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MATHÉMATIQUES

QUELQUES REMARQUES SUR LE THÉORÈME LIMITE LIAPOUNOFF

Par SERGE BERNSTEIN, de l'Académie

1. Définitions. Soit $\sum_1^n x_i = S_n$ une somme de grandeurs aléatoires indépendantes $EMx_i = 0$, $EMx_i^2 = b_i$, $EM|x_i|^p = c_{i,p}$, $B_n = \sum_1^n b_i$.

Nous donnerons à $M_{n,p} = \dfrac{\sum_1^n c_{i,p}}{B_n^{\frac{p}{2}}}$ le nom de paramètre de Liapounoff d'ordre p et nous appellerons la condition

$$\lim_{n=\infty} M_{n,p} = 0 \tag{1}$$

condition de Liapounoff d'ordre p. Nous dirons que le théorème limite d'ordre $p > 2$ est applicable à S_n, si l'on a

$$\text{prob.}\left(\frac{S_n}{\sqrt{B_n}} < t\right) \to \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{t} e^{-\frac{t^2}{2}} dt = G(t) \tag{2}$$

(uniformément pour $-\infty < t < \infty$)

et en même temps

$$\lim_{n=\infty} EM\left|\frac{S_n}{\sqrt{B_n}}\right|^p = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |t|^p e^{-\frac{t^2}{2}} dt = R_p. \tag{3}$$

Théorème. *La condition d'ordre p de Liapounoff est nécessaire et suffisante pour que le théorème limite d'ordre p soit applicable à* $S_n = \sum_1^n x_i$, *si les grandeurs* x_i *sont uniformément négligeables* par rapport à* S_n $(i \leqslant n)$.

* Les grandeurs x_i $(i \leqslant n)$ sont dites uniformément négligeables par rapport à S_n, si la différence entre les probabilités des inégalités $S_n - x_i < t\sqrt{B_n} = z$ et $S_n < t\sqrt{B_n} = z$ tend uniformément vers zéro pour $n \to \infty$ (quel que soit z); actuellement cette condition est équivalente à $\dfrac{\max_{i \leqslant n} b_i}{B_n} \to 0$. Dans la suite dans l'affirmation de la nécessité d'une condition nous supposons toujours que les grandeurs x_i sont uniformément négligeables.

Supposons d'abord $p \geqslant 2$ entier et pair. Dans ce cas notre affirmation résultera du calcul direct de $EM\left|\frac{S_n}{\sqrt{B_n}}\right|^p$ effectué par A. A. Markoff qui met en évidence que la grandeur

$$\varphi_{M_{n,p}} = EM\left|\frac{S_n}{\sqrt{B_n}}\right|^p - R_p \tag{4}$$

tend vers zéro avec $M_{n,p}$, et de plus, si $M_{n,p} > M > 0$ et $\frac{\max_{i \leqslant n} b_i}{B_n} \to 0$ pour $n \to \infty$, on a $\varphi_{M_{n,p}} > \lambda_p > 0$. En effet, d'après le théorème de Liapounoff, la condition (1) entraîne (2) et l'égalité (4) donne (3), à cause de (1). Inversement, si l'on avait $M_{n,p} > M > 0$, l'égalité (3) serait impossible à cause de la seconde propriété de la grandeur $\varphi_{M_{n,p}}$.

Avant de passer au cas de $p > 2$ quelconque observons que d'après (4) la condition de Liapounoff d'ordre p pair est équivalente à la seule condition (3) du même ordre p qui peut d'ailleurs être remplacée par la condition encore moins restrictive

$$\lim_{n=\infty} EM\left|\frac{S_n}{\sqrt{B_n}}\right|^p \leqslant R_p, \tag{3bis}$$

car le signe d'inégalité dans (3bis) est impossible (dans l'hypothèse de la négligeabilité des grandeurs x_i). De plus, il est facile à déduire de (4) que si la loi des probabilités de $\frac{S_n}{\sqrt{B_n}}$ tend vers une limite $F(t)$ différente de $G(t)$, tous les moments d'ordres pairs $p > 2$ relatifs à $F(t)$ sont supérieurs aux moments correspondants de $G(t)$, d'où il résulte, en particulier, que la fonction charactéristique $\Theta(z)$ de $F(t)$ ou bien n'est pas entière, ou bien on a $\frac{\Theta(iy) + \Theta(-iy)}{2} > e^{\frac{y^2}{2}}$ pour toute valeur réelle de y.

2. Nous aurons besoin des deux lemmes suivants dont la démonstration ne présente pas de difficultés.

Lemme I. *Si les lois des probabilités* $F_n(x)$ des grandeurs y_n tendent uniformément vers $F(x)$

$$|F_n(x) - F(x)| < \varepsilon_n \qquad (-\infty < x < \infty)$$

et

$$\int_{-\infty}^{\infty} |x|^p dF_n(x) < c$$

et

$$\int_{-\infty}^{\infty} |x|^p dF(x) < c$$

pour une certaine valeur $p > 0$, on a pour toute valeur positive de $q < p$

$$\int_{-\infty}^{\infty} |x|^q dF_n(x) \to \int_{-\infty}^{\infty} |x|^q dF(x)$$

lorsque $\varepsilon_n \to 0$ *(uniformément pour* $q \leqslant p' < p$*).*

Lemme II. *Soit* $S_n = \sum_1^n x_i$, $S'_n = \sum_1^n x'_i$, *les lois des probabilités de* x'_i *et* x_i *étant identiques et toutes les grandeurs* x_i, x'_k *étant indépendantes.*

Dans ces conditions: 1) *le théorème limite d'ordre p est en même temps applicable* (ou non applicable) *aux sommes* S_n *et* $S_n - S'_n = \sum_1^n (x_i - x'_i)$; 2) *les paramètres de Liapounoff du même ordre p de* S_n *et* $S_n - S'_n$ *sont en même temps bornés et en même temps tendent vers zéro.*

Grâce au lemme II nous pouvons nous borner au cas où toutes les grandeurs x_i sont soumises à des lois de probabilité symétrique.

Dans cette hypothèse, on a, en posant $S_n^{(i)} = S_n - x_i$, $\delta = p-2 > 0$,

$$EM\,|S_n|^{\delta+2} = EM\sum_1^n [x_i^2\,|S_n|^\delta + x_i S_n^{(i)}\,|S_n|^\delta] =$$

$$= \frac{1}{2}\,EM\sum_1^n x_i^2\,[(|S_n^{(i)}|+|x_i|)^\delta + \big||S_n^{(i)}|-|x_i|\big|^\delta] +$$

$$+ \frac{1}{2}\,EM\,[(|S_n^{(i)}|+|x_i|)^\delta - \big||S_n^{(i)}|-|x_i|\big|^\delta].$$

Donc

$$EM\,|S_n|^{\delta+2} - EM\,(\delta+1)\sum_1^n x_i^2\,|S_n^{(i)}|^\delta = EM\sum_1^n T_i, \tag{5}$$

où

$$T_i = \frac{1}{2}\,\{[x_1^2 + |x_i S_n^{(i)}|]\,[|S_n^{(i)}|+|x_i|]^\delta +$$

$$+ [x_i^2 - |x_i S_n^{(i)}|]\,\big||S_n^{(i)}|-|x_i|\big|^\delta\} - (\delta+1)\,x_i^2\,|S_n^{(i)}|^\delta.$$

Ainsi, pour $|x_i| \geqslant |S_n^{(i)}|$, on a

$$T_i = \frac{1}{2}\,|x_i|\,\{[|x_i|+|S_n^{(i)}|]^{\delta+1} + [|x_i|-|S_n^{(i)}|]^{\delta+1} -$$

$$- 2\,(\delta+1)\,|x_i|\,|S_n^{(i)}|^\delta\}, \tag{6}$$

et, pour $|x_i| \leqslant |S_n^{(i)}|$, on a

$$T_i = \frac{1}{2}\,|x_i|\,\{[|S_n^{(i)}|+|x_i|]^{\delta+1} - [|S_n^{(i)}|-|x_i|]^{\delta+1} -$$

$$- 2\,(\delta+1)\,|x_i|\,|S_n^{(i)}|^\delta\}. \tag{6bis}$$

Donc, dans le premier cas ($|x_i| \geqslant |S_n^{(i)}|$)

$$|T_i| < 2^\delta\,|x_i|^{\delta+2} \tag{7}$$

et de plus, si $\delta \geqslant 2$

$$T_i > \frac{1}{2}\,|x_i|^{\delta+2}. \tag{8}$$

Dans le second cas ($|x_i| \leqslant |S_n^{(i)}|$), on tire de (6bis)

$$\left.\begin{aligned} &|T_i| < 2^\delta\,|x_i|^{\delta+2} \qquad (0 < \delta \leqslant 2) \\ &|T_i| < \frac{(\delta+1)\,\delta\,(\delta-1)\,2^\delta}{24}\,|x_i|^4\,|S_n^{(i)}|^{\delta-2} \quad (\delta \geqslant 2) \end{aligned}\right\} \tag{7bis}$$

et, de plus, si $\delta \geqslant 2$

$$T_i > \frac{(\delta+1)\,\delta\,(\delta-1)}{12}\,|x_i|^4\,|S_n^{(i)}|^{\delta-2} \geqslant \frac{(\delta+1)\,\delta\,(\delta-1)}{12}\,|x_i|^{\delta+2}. \tag{8bis}$$

Donc, d'après (5), (7) et (7bis), on a pour $0 < \delta = p-2 \leqslant 2$

$$\left|EM\left|\frac{S_n}{\sqrt{B_n}}\right|^p - (p-1)\sum_1^n \frac{b_i}{B_n}\,EM\left|\frac{S_n^{(i)}}{\sqrt{B_n}}\right|^{p-2}\right| < 2^{p-2} M_{n,p}. \tag{9}$$

Supposons à présent que la condition $M_{n,p} \to 0\,(2 < p \leqslant 4)$ est remplie. D'après le théorème de Liapounoff et le lemme I, on a

$$\lim_{n=\infty} EM \left| \frac{S_n^{(i)}}{\sqrt{B_n}} \right|^{p-2} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} |t|^{p-2} e^{-\frac{t^2}{2}} dt = R_{p-2} = \frac{R_p}{p-1}.$$

Il résulte donc de (9) que

$$\lim_{n=\infty} \left| EM \frac{S_n}{\sqrt{B_n}} \right|^p - R_p \Big| \leqslant \lim_{n=\infty} 2^{p-2} M_{n,p} = 0. \tag{10}$$

Si $p > 4$, il suffit d'observer que $M_{n,p} \to 0$ entraîne $M_{n,p} \to 0$ pour $2 < p' < p$, de sorte qu'en admettant que (10) est déjà démontré pour $p-2$ et en tenant compte de l'inégalité

$$|T_i| < 2^{\delta} \left[|x_i|^{\delta+2} + \frac{\delta^3}{24} x_i^4 |S_n^{(i)}|^{\delta-2} \right] \quad (\delta > 2)$$

qui résulte de (7) et (7^{bis}), nous aurons également

$$\lim_{n=\infty} \left| EM \left| \frac{S_n}{\sqrt{B_n}} \right|^p - R_p \right| \leqslant \lim_{n=\infty} 2^{p-2} \left[M_{n,p} + \frac{(p-2)^3}{24} M_{n,4} R_{p-2} \right] = 0. \tag{10^{bis}}$$

Pour démontrer la réciproque, remarquons que, d'après (8) et (8^{bis}), on a pour $p \geqslant 4$

$$T_i \geqslant \frac{1}{2} |x_i|^p;$$

donc

$$EM \left| \frac{S_n}{\sqrt{B_n}} \right|^p - (p-1) \sum_1^n \frac{b_i}{B_n} EM \left| \frac{S_n^{(i)}}{\sqrt{B_n}} \right|^{p-2} > \frac{1}{2} M_{n,p}$$

pour $p \geqslant 4$; par conséquent (dans l'hypothèse que les grandeurs x_i sont négligeables) la condition de Liapounoff $M_{n,p} \to 0$ est équivalente à la condition

$$EM \left| \frac{S_n}{\sqrt{B_n}} \right|^p - (p-1) EM \left| \frac{S_n}{\sqrt{B_n}} \right|^{p-2} \to 0, \quad (p \geqslant 4),$$

donc le théorème limite d'ordre $p \geqslant 4$ ne peut pas être applicable, si $M_{n,p}$ ne tend pas vers zéro. Cette démonstration de la réciproque ne s'étend pas cependant au cas de $2 < p < 4$. Il convient donc d'indiquer encore une autre démonstration.

Posons

$$EM |S_n|^{\delta+2} = EM |S_n'|^{\delta+2} + \sum^*,$$

où la somme $S_n' = \sum_1^n x_i'$ contient toutes les valeurs $x_i = x_i'$ satisfaisant à l'inégalité $|x_i| \leqslant \tau \sqrt{B_n}$, où $\tau > 0$ est une constante arbitrairement petite. Soit

$$b_i' = EM\, x_i'^2, \quad c_{i,p}' = EM |x_i'|^p, \quad c_{i,p}^* = c_{i,p} - c_{i,p}'.$$

On a alors

$$\sum^* > \sum_1^n \sum_{|x_i| > \tau\sqrt{B_n}} EM\, x_i^2 |S_n|^{\delta} \geqslant \frac{1}{2} \sum_1^n \sum_{|x_i| > \tau\sqrt{B_n}} EM\, x_i^2 [|x_i| + |S_n^{(i)}|]^{\delta} > \frac{1}{2} \sum_1^n c_{i,p}^*.$$

Or, on a manifestement $c'_{i,p} < b'_i(\tau\sqrt{B_n})$; donc, en tenant compte du fait connu que $\lim_{n=\infty} \frac{\sum_1^n b'_i}{B_n} = 1$, lorsque le théorème limite d'ordre $p \geqslant 2$ est applicable, on a

$$\frac{\sum_1^n c'_{i,p}}{\left[\sum_1^n b'^{p}_{i,p}\right]^{\frac{p}{2}}} < \left[\tau\sqrt{\frac{B_n}{\sum_1^n b'_i}}\right]^{\delta} < 2\tau^{\delta}$$

pour n suffisamment grand. Par conséquent, en appliquant (10) et (10bis) à $\frac{S'_n}{\sqrt{\sum_1^n b'_i}}$, on a

$$\frac{\sum^*}{B_n^{\frac{p}{2}}} = \left|\frac{EM|S_n|^p - EM|S'_n|^p}{B_n^{\frac{p}{2}}}\right| < \varepsilon_{n,\tau}, \qquad (11)$$

où $\varepsilon_{n,\tau}$ est aussi petit qu'on veut pour n assez grand et τ assez petit d'où il résulte que, le théorème limite d'ordre p étant applicable,

$$M_{n,p} = \sum_1^n \frac{c'_{i,p} + c^*_{i,p}}{B_n^{\frac{p}{2}}} < 2(\tau^{\delta} + \varepsilon_{n,\tau})$$

doit tendre vers zéro avec $n \to \infty$.

3. Remarque. La condition de M. Lindberg $\lim_{n=\infty} \frac{\sum_1^n b'_i}{B_n} = 1$ que nous venons d'utiliser est nécessaire et suffisante pour l'applicabilité du théorème limite d'ordre * $p=2$. Mais, comme je l'ai indiqué en 1926 dans mon Mémoire (1) «Sur l'extension du théorème limite du calcul des probabilités aux sommes de quantités dépendantes», le théorème limite peut être applicable même dans des cas où B_n n'existe pas (pages 12 et 23). Voici le théorème que j'ai donné à l'endroit cité **: «ε_k désignant la probabilité que $|x_k| > L$, le théorème limite sera applicable toutes les fois que l'on a (en choisissant convenablement L) en même temps $\frac{L^2}{B_n} \to 0$ et $\sum_1^n \varepsilon_k \to 0$, la valeur de B_n étant calculée dans l'hypothèse que $|x_k| \leqslant L$». Plus tard (en 1935) M. Feller (2), sans connaître mon théorème, le retrouva et démontra de plus que la condition énoncée est aussi nécessaire (si zéro est la médiane commune des grandeurs x_k). Ainsi, lorsque ma condition est remplie, la dispersion B'_n, calculée (raccourcie)

---

* D'après ce qui précède, la condition (11) $\frac{EM|S_n|^p - EM|S'_n|^p}{B_n^{\frac{p}{2}}} \to 0$ qui est analogue à la condition de M. Lindberg est aussi nécessaire et suffisante pour l'applicabilité du théorème limite d'ordre $p \geqslant 2$.

** Page 23 je l'ai étendu aux sommes de grandeurs presque indépendantes.

par le moyen indiqué, peut être inférieure à B_n même lorsque B_n existe, et $p<2$, si $\frac{B_n'}{B_n}<1$. D'après le lemme I, si $\frac{B_n'}{B_n}>\lambda>0$, l'ordre $p<2$ du théorème limite est aussi voisin de 2 que l'on veut; et dans ce cas $EM\left|\frac{S_n}{\sqrt{B_n'}}\right|\to R_p$ pour toute valeur de $p<2$.

Lorsque B_n n'existe pas, le maximum * $p_0\leqslant 2$ (atteint ou non) des ordres d'applicabilité du théorème limite qui est égal au maximum des valeurs de p, pour lesquelles $EM\left|\frac{S_n}{\sqrt{B_n'}}\right|^p$ est borné, peut prendre toutes les valeurs entre 0 et 2. La condition nécessaire et suffisante pour que le théorème limite d'un ordre donné $p<2$ soit applicable peut être donnée sous une forme analogue aux conditions de Liapounoff.

Théorème. *Soit* $\tau=\frac{L}{\sqrt{B_n'}}$ *un nombre arbitrairement petit, où* B_n' *est la dispersion raccourcie, calculée dans l'hypothèse que* $|x_k|\leqslant L$; *soit* $c_{k,p}^*=EM|x_k|^p$ *se rapportant seulement aux valeurs* $|x_k|>L$; *cela posé, pour que le théorème limite d'ordre* $p>0$ *soit applicable, il faut et il suffit que*

$$\frac{\sum_1^n c_{k,p}^*}{(\sqrt{B_n'})^p}\to 0$$

(pour la nécessité on suppose les grandeurs x_k négligeables et rapportées à leur médiane).

Manuscrit reçu
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LITTÉRATURE CITÉE

[1] S. Bernstein, Math. Ann., 97 (1926). [2] W. Feller, Math. ZS., 40 (1935).

* Dans l'exemple que j'ai donné à l'endroit cité l'ordre p du théorème limite est aussi voisin qu'on veut de 2; la valeur de $B_n'\sim\frac{6n\log n}{\pi}$, donc, par exemple, $EM|S_n|\sim R_1\sqrt{B_n'}\sim\frac{2}{\pi}\sqrt{3n\log n}$.

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MATHEMATICS

SOME PROPERTIES OF THE DECOMPOSITIONS OF ABELIAN GROUPS WITHOUT TORSION INTO DIRECT SUMS

By EUGENE LIAPIN

(Communicated by I. M. Vinogradow, Member of the Academy, 4. V. 1939)

In this paper we shall consider some properties of Abelian groups without torsion, i. e. commutative groups having no element of finite order (except 0). Since we will not consider any other groups, by a «group» we shall in the sequel always mean an Abelian group without torsion. The operation of composition of elements of a group we shall indicate by $+$.

The results of this paper may be, perhaps, of some interest by themselves and, besides, they will be used in my forthcoming paper «On the decomposition of countable Abelian groups without torsion into direct sums of groups of the first rank» which will appear in the same Journal. The results of this paper are given with sketches of proofs only. Detailed proofs of the theorems announced in this paper will be given elsewhere.

§ 1. As a preliminary we shall consider sets α of natural numbers possessing the following properties: 1) if $m \in \alpha$ and $n \in \alpha$ then the least common multiple of m and n also belongs to α; 2) if $m \in \alpha$, then every divisor of m also belongs to α. In the sequel such sets will be called characteristics. We shall say that a characteristic β is divisible by another characteristic α $\left(\text{in symbols } \frac{\alpha}{\beta}\right)$ if $\alpha \subset \beta$. The common part of a set of characteristics $\{\alpha_\tau\}$ is obviously also a characteristic; it is the greatest common divisor of the characteristics α_τ, i. e. the common divisor which is divisible by any other common divisor of these characteristics. If α is a characteristic and n a natural number, then we shall call the product of n and α (and denote by $n\alpha$) the set of all numbers of the form $m\alpha$, where m/n and $a \in \alpha$. It is evidently a characteristic.

We shall say that two characteristics α and β are equivalent (in symbols $\alpha \sim \beta$) if there exist two natural numbers m and n such that $m\alpha = n\beta$. The property of equivalence is evidently reflexive, symmetric and transitive, so that all characteristics can be arranged into classes of equivalent characteristics. We shall call these classes overtypes. We shall say that an overtype $\overline{\alpha}$ is a divisor of another overtype $\overline{\beta}$ $\left(\text{and write } \frac{\overline{\alpha}}{\overline{\beta}}\right)$ if there exist characteristics $\alpha \in \overline{\alpha}$ and $\beta \in \overline{\beta}$ such that $\frac{\alpha}{\beta}$. If $\frac{\overline{\alpha}}{\overline{\beta}}$ and $\overline{\alpha} \neq \overline{\beta}$, we shall say that $\overline{\alpha}$ is a proper divisor of $\overline{\beta}$.

It may be proved without difficulty that: 1) $\frac{\overline{\alpha}}{\beta}$ and $\frac{\overline{\beta}}{\gamma}$ implies $\frac{\overline{\alpha}}{\gamma}$; 2) $\frac{\overline{\alpha}}{\beta}$ and $\frac{\overline{\beta}}{\alpha}$ implies $\overline{\alpha}=\overline{\beta}$; 3) if $\overline{\alpha}_1, \ldots \overline{\alpha}_n$ are overtypes and $\alpha_1 \in \overline{\alpha}_1, \ldots$ $\ldots \alpha_n \in \overline{\alpha}_n$ are characteristics belonging to these overtypes, then the overtype determinated by the greatest common divisor of $\alpha_1, \ldots \alpha_n$ is the greatest common divisor of $\overline{\alpha}_1, \ldots \overline{\alpha}_n$ (in the same sense as above).

§ 2. Let $\mathfrak{G}$ be a group. Any subgroup $\mathfrak{H}$ of $\mathfrak{G}$ such that $nX \in \mathfrak{H}$ (where n is an integer and X is an element of $\mathfrak{G}$) implies $X \in \mathfrak{H}$ will be called a non-extensible subgroup of $\mathfrak{G}$. Obviously $\mathfrak{G}$ is its own non-extensible subgroup and the intersection (common part) of a set of non-extensible subgroups of $\mathfrak{G}$ is itself a non-extensible subgroup of $\mathfrak{G}$. Let $\mathfrak{A}$ be a subset of $\mathfrak{G}$ (not necessarily a subgroup); the least non-extensible subgroup of $\mathfrak{G}$ containing $\mathfrak{A}$ will be called the extension of $\mathfrak{A}$ and denoted by $\mathfrak{R}(\mathfrak{A})$. A direct summand of $\mathfrak{G}$ is a non-extensible subgroup. The extension of a set consisting of a single element is a group of the first rank.

Let $\mathfrak{A}$ be a subset of $\mathfrak{G}$. If there exists such an element $X \in \mathfrak{G}$ that $nX \in \mathfrak{A}$, we shall say that $\mathfrak{A}$ is divisible by n. Let $\mathfrak{A}=G+\mathfrak{H}$ be a coset (where $\mathfrak{H}$ is a subgroup of $\mathfrak{G}$ and $G \in \mathfrak{G}$); then the set of all divisors of $\mathfrak{A}$ (i. e. of all natural numbers by which $\mathfrak{A}$ is divisible) is a characteristic. We shall call this set the characteristic of $\mathfrak{A}$ and denote it by $\chi(\mathfrak{A})$. The overtype to which the characteristic $\chi(\mathfrak{A})$ belongs will be called the overtype of $\mathfrak{A}$ and denoted by $\sigma(\mathfrak{A})$ (observe that $\mathfrak{A}$ may contain a single element).

If $\mathfrak{H}$ is a subgroup of $\mathfrak{G}$ which contains an element X with the overtype $\sigma(\sigma(X)=\sigma)$, then we shall say that σ is an overtype belonging to $\mathfrak{H}$. We shall call the element $Y \in \mathfrak{A}$ (where $\mathfrak{A}$ is a subset of $\mathfrak{G}$) a superior element of $\mathfrak{A}$, if $\sigma(Y)$ is not a proper divisor of the overtype of any element belonging to $\mathfrak{A}$.

Let $\mathfrak{H}$ be a subgroup of $\mathfrak{G}$ and σ an overtype belonging to $\mathfrak{G}$. We shall consider in the sequel the following non-extensible subgroups of $\mathfrak{G}$: 1) $\mathfrak{H}(\sigma$ — the extension of the set of all elements $X \in \mathfrak{H}$, whose overtypes $\sigma(X)$ are not proper divisors of σ; 2) $\overline{\mathfrak{H}}(\sigma)$ — the extension of the set of all elements $X \in \mathfrak{H}$ whose overtypes $\sigma(X)$ are not divisors of σ; 3) $\mathfrak{H}(\sigma)$ — the group of all elements of $\mathfrak{H}$ whose overtypes are divisible by σ. An element $X \in \mathfrak{H}$ satisfying the condition $\chi(X)=\chi\left(X+\overline{\mathfrak{H}}(\sigma(X))\right)$ will be called regular in $\mathfrak{H}$.

Observe that the condition $\sigma(A+\mathfrak{A})=\sigma(B+\mathfrak{B})$, where $\mathfrak{A}$ and $\mathfrak{B}$ are two non-extensible subgroups of $\mathfrak{G}$ and $A, B \in \mathfrak{G}$ is equivalent to the existence of an isomorphism: $\mathfrak{R}(A+\mathfrak{A}) \mid \mathfrak{A} \cong \mathfrak{R}(B+\mathfrak{B}) \mid \mathfrak{B}$ (the ranks of these groups are equal to 1) and the condition $\chi(A+\mathfrak{A})=\chi(B+\mathfrak{B})$ is equivalent to the existence of such an isomorphism between the above subgroups in which the coset $A+\mathfrak{A}$ corresponds to the coset $B+\mathfrak{B}$.

§ 3. Lemma 1. *Let $\mathfrak{G}$ be a direct sum: $\mathfrak{G}=\mathfrak{A} \oplus \mathfrak{B} \oplus \ldots$ and let $X=A+B+\ldots$ where $A \in \mathfrak{A}$, $B \in \mathfrak{B}, \ldots$ Then $\chi(X)$ is the greatest common divisor of the characteristics $\chi(A), \chi(B), \ldots$*

Lemma 2. *Let $\mathfrak{H}$ be a non-extensible subgroup of $\mathfrak{G}$ such that the rank $\left(\frac{\mathfrak{G}}{\mathfrak{H}}\right)=1$ and that there exists an element $X \overline{\in} \mathfrak{H}$ with $\chi(X)=\overline{=}\chi(X+\mathfrak{H})$. Then $\mathfrak{G}$ is the direct sum of $\mathfrak{H}$ and $\mathfrak{R}(X)$.*

Let $\mathfrak{H}$ be again a non-extensible subgroup of $\mathfrak{G}$ such that the rank $\left(\frac{\mathfrak{G}}{\mathfrak{H}}\right)=1$ and denote by F the set of all elements of $\mathfrak{G}$ not contained

in $\mathfrak{H}$. We shall say that an element $X \in \mathfrak{G}$ satisfies the condition $[X, \mathfrak{H}]$. if the following equalities hold:

$$\sigma(X) = \sigma(X + \mathfrak{H}); \ \chi(X + \mathfrak{H}) = \chi\left(X + \tilde{\mathfrak{H}}(\sigma(X))\right).$$

The following theorem gives a necessary and sufficient condition for $\mathfrak{H}$ to be a direct summand of $\mathfrak{G}$.

Theorem 1. a) *If $\mathfrak{H}$ is a direct summand of $\mathfrak{G}$ (and $\mathfrak{H} \neq \mathfrak{G}$), than the set of superior elements of F is not void and every such element X satisfies the condition* $[X, \mathfrak{H}]$.

b) *If $\mathfrak{H}$ is not a direct summand of $\mathfrak{G}$, then F contains no element satisfying the condition* $[X, \mathfrak{H}]$.

Proof. a) Let $\mathfrak{G} = \mathfrak{H} \oplus \mathfrak{S}$. Then any element X of the form $X = S + H$, where $S \in \mathfrak{S}$, $S \neq 0$ and $H \in \tilde{\mathfrak{H}}(\sigma(S))$, is a superior element of F and vice versa. From lemma 1 we easily obtain: $\chi(X) \sim \chi(S)$, $\chi(X + \mathfrak{H}) = \chi(S)$, $\chi\left(X + \tilde{\mathfrak{H}}(\sigma(X))\right) = \chi(S)$, whence follows immediately that X satisfies the condition $[X, \mathfrak{H}]$.

b) Suppose that F contains an element X satisfying the condition $[X, \mathfrak{H}]$. Then using the general properties of the characteristics, we can construct such an element $Y \in F$ that $\chi(Y) = \chi(Y + \mathfrak{H})$. By lemma 2 $\mathfrak{H}$ is then a direct summand of $\mathfrak{G}$.

Theorem 2. *Let $\mathfrak{G}$ be a group decomposible into a direct sum of groups of the first rank, and X a non-zero element of $\mathfrak{G}$. Then, in order that $\mathfrak{R}(X)$ should be a direct summand in one of such decompositions, it is necessary and sufficient that the element X should be regular in $\mathfrak{G}$.*

Proof. Necessity. We can find such a decomposition of $\mathfrak{G}$ into a direct sum that $\mathfrak{R}(X)$ and $\overline{\mathfrak{G}}(\sigma(X))$ are its summands. Then the regularity becomes evident.

Sufficiency. Let $\mathfrak{G} = \mathfrak{S}_1 \oplus \mathfrak{S}_2 \oplus \ldots \oplus \mathfrak{S}_n \oplus \mathfrak{S}_\alpha \oplus \mathfrak{S}_\beta \oplus \ldots$ where the rank $\mathfrak{S}_\nu = 1$ $(\nu = 1, 2, \ldots n, \alpha, \beta, \ldots)$ and $X = S_1 + S_2 + \ldots + S_n$, $S_i \in \mathfrak{S}_i$ $(i = 1, 2, \ldots n)$. Consider now the following subgroups of $\mathfrak{G}$: $\mathfrak{H}_0 = \overline{\mathfrak{G}}(\sigma(X))$, $\mathfrak{H}_1 = \mathfrak{R}(\mathfrak{H}_0, X)$, $\mathfrak{H}_{k+1} = \mathfrak{R}(\mathfrak{H}_k, S_k)$ $(k = 1, 2, \ldots n)$. If $\mathfrak{H}_{k+1} \neq \mathfrak{H}_k$, then the rank $\frac{\mathfrak{H}_{k+1}}{\mathfrak{H}_k} = 1$. Hence, by lemma 2 ($X$ being a regular element), $\mathfrak{H}_1 = \mathfrak{H}_0 \oplus \mathfrak{R}(X)$ and by theorem 1 $\mathfrak{H}_k$ is a direct summand of $\mathfrak{H}_{k+1}$ $(k = 1, 2, \ldots n-1)$. Hence we may easily deduce the existence of the decomposition in question.

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MATHEMATICS

ON THE DECOMPOSITION OF COUNTABLE ABELIAN GROUPS WITHOUT TORSION INTO DIRECT SUMS OF GROUPS OF THE FIRST RANK

By EUGENE LIAPIN

(Communicated by I. M. Vinogradow, Member of the Academy, 4. V. 1939)

In this paper we shall give a necessary and sufficient condition for the decomposibility of a countable Abelian group into a direct sum of groups of the first rank. We retain the terminology and the denotations of my paper «Some properties of the decompositions of Abelian groups without torsion into direct sums» (this Journal, this issue) the results of which are supposed to be known. Detailed proofs of theorems announced in the present paper will be given elsewhere.

§ 1. In the sequel by $\mathfrak{G}$ we shall always denote an Abelian group without torsion. Let σ be an overtype belonging to $\mathfrak{G}$ and consider the set consisting of the zero element and of all regular elements having σ for their overtype. If $\mathfrak{G}$ is the sum of all such sets we shall say that $\mathfrak{G}$ admits of a regular representation. The representation $G = G_1 + G_2 + \ldots + G_m$ of an element $G \in \mathfrak{G}\,(G \neq 0)$ as a sum of regular elements having different overtypes will be called a regular representation of G and the number m of summands in such a representation will be called the length of the representation. For the zero element 0 of $\mathfrak{G}$ the regular representation will be $0 = \bar{0}$ and its length is equal to 0. In order that $\mathfrak{G}$ should admit a regular representation it is obviously necessary and sufficient that all elements of $\mathfrak{G}$ should admit a regular representation.

We shall say that a set $\Omega = (\mathfrak{A}, \mathfrak{B}, \ldots \mathfrak{N}$ of non-extensible subgroups of $\mathfrak{G}$ satisfies the condition [*] if to every subgroup $\mathfrak{N} \in \Omega$ we can correlate an overtype $\sigma[\mathfrak{N}]$ belonging to $\mathfrak{G}$ in such a way that the following conditions are satisfied: 1) $\mathfrak{N}$ can be represented as a direct sum of groups of the first rank; 2) $\mathfrak{G}(\sigma[\mathfrak{N}]) = \mathfrak{N} \oplus \overline{\mathfrak{G}}(\sigma[\mathfrak{N}])$; 3) to different elements of Ω correspond different overtypes: $\mathfrak{N} \neq \mathfrak{M}$ implies $\sigma(\mathfrak{N}) \neq \sigma(\mathfrak{M})$.

By $\mathfrak{G}[\Omega]$ we shall denote the least subgroup of $\mathfrak{G}$ containing all the subgroups belonging to Ω.

We shall now introduce new objects (which will be called in the sequel simply «objects»), viz. ordered sets of three elements (Ω, G, X), where Ω is a finite set of subgroups of $\mathfrak{G}$ satisfying the condition [*] and G and X are two elements of $\mathfrak{G}$ such that $G - X \in \mathfrak{G}[\Omega]$. If α is an «object», then its elements we shall denote resp. by $\Omega(\alpha)$, $G(\alpha)$ and $X(\alpha)$.

To every «object» α correspond two non-negative integers $a(\alpha)$ and $m(\alpha)$, where $a(\alpha)$ is the number of groups $\mathfrak{N} \in \mathfrak{Q}(\alpha)$ such that $\overline{\mathfrak{G}}(\sigma[\mathfrak{N}])$ does not contain $X(\alpha)$ and $m(\alpha)$ is the minimal length of a regular representation of $X(\alpha)$. We shall say that an «object» α precedes another «object» β ($\alpha < \beta$) if the following three conditions are satisfied: 1) $G(\alpha) = G(\beta)$; 2) $\mathfrak{Q}(\beta) \subset \mathfrak{Q}(\alpha)$; 3) either $a(\alpha) < a(\beta)$, or $a(\alpha) = a(\beta)$ and $m(\alpha) < m(\beta)$. Observe that a chain of decreasing «objects» $\alpha_1, \alpha_2 \ldots$ $\ldots (\alpha_1 > \alpha_2 > \ldots)$ is always finite.

§ 2. Lemma 1. *Let $\mathfrak{G}$ be a group admitting a regular representation and possessing the following property: for any overtype σ belonging to $\mathfrak{G}$ the group $\mathfrak{G}(\sigma)$ is decomposible into a direct sum of the group $\overline{\mathfrak{G}}(\sigma)$ and of groups of the first rank. Than, whatever be the «object» α such that $m(\alpha) \neq 0$ (i. e. $X(\alpha) \neq 0$), there exists an «object» β which precedes α ($\beta < \alpha$).*

Proof. Consider a regular representation of $X(\alpha)$: $X(\alpha) = X_1 + X_2 + \ldots + X_m$ of minimal length. We can suppose the overtype $\sigma(X_1) = \sigma_1$ to be not divisible by overtypes of other elements X_i. Two cases are now possible.

1) There exists such an $\mathfrak{N} \in \mathfrak{Q}(\alpha)$ that $\sigma[\mathfrak{N}] = \sigma_1$. Then $X_1 = U + N$, where $U \in \overline{\mathfrak{G}}(\sigma_1)$, $N \in \mathfrak{N}$. Consider the «object» $\beta = (\mathfrak{Q}(\alpha), G(\alpha), X(\alpha) - N)$. In order to prove that β precedes α, it suffices to show that $a(\beta) < a(\alpha)$. But this inequality follows from the fact that $X(\alpha) \overline{\in} \overline{\mathfrak{G}}(\sigma_1)$, while $X(\beta) = X(\alpha) - N \in \overline{\mathfrak{G}}(\sigma_1)$ and, on the other hand, from the fact that there cannot exist such an overtype σ_2 that $X(\alpha) \in \overline{\mathfrak{G}}(\sigma)$, while $X(\alpha) - N \in \overline{\mathfrak{G}}(\delta_2)$ (this can be proved using the regularity of X_1).

2) There exists no such $\mathfrak{N} \in \mathfrak{Q}(\alpha)$ that $\sigma[\mathfrak{N}] = \sigma_1$. Using theorem 2 of the paper «Some properties etc.» referred to above we can prove that there exists a decomposition $\mathfrak{G}(\sigma_1) = \mathfrak{M} \oplus \overline{\mathfrak{G}}(\sigma_1)$ such that $X_1 \in \mathfrak{M}$. Consider the «object» $\beta = ((\mathfrak{Q}(\alpha), \mathfrak{M}), G(\alpha), X(\alpha) - X_1)$, where $\sigma[\mathfrak{M}] = \sigma_1$. It is evident that $m(\beta) < m(\alpha)$. We can see as above that $a(\beta) \leqslant a(\alpha)$ and thus $\beta < \alpha$.

Lemma 2. *Let $\mathfrak{G}$ be a group satisfying all the conditions of lemma 1, G be an element of $\mathfrak{G}$ and $\mathfrak{Q}_1$ be a finite set of subgroups of $\mathfrak{G}$ satisfying the condition* [*]. *Then there exists a finite set $\mathfrak{Q}_2$ of subgroups of $\mathfrak{G}$ satisfying the condition* [*] *and such that $\mathfrak{Q}_1 \subset \mathfrak{Q}_2$ and $G \in \mathfrak{G}[\mathfrak{Q}_2]$.*

Proof. Denote by α_1 the «object» $(\mathfrak{Q}_1, G, G)$. By lemma 1, if $m(\alpha_1) \neq 0$, there exists an $\alpha_2 < \alpha_1$ and in the same way [if $m(\alpha_2) \neq 0$] an $\alpha_3 < \alpha_2$. The chain $\alpha_1 > \alpha_2 > \ldots$ must be finite so that we arrive at last at the «object» $\alpha_t \leqslant \alpha_1$ with $m(\alpha_t) = 0$, i. e. $\alpha_t = (\mathfrak{Q}_2, G, O)$ where $\mathfrak{Q}_1 \subset \mathfrak{Q}_2$ and $G = G - O \in \mathfrak{G}[\mathfrak{Q}_2]$.

§ 3. Suppose now that the group $\mathfrak{G}$ is countable. If σ is an overtype belonging to $\mathfrak{G}$, we enumerate all the elements of $\mathfrak{G}$ having for their overtypes $G_{\sigma,1}, G_{\sigma 2}, \ldots$ and define a sequence of non-extensible subgroups of $\mathfrak{G}$:

$$\mathfrak{G}_{\sigma,0} = \overline{\mathfrak{G}}(\sigma), \quad \mathfrak{G}_{\sigma,n} = \mathfrak{R}(\mathfrak{G}_{\sigma,n-1}, G_{\sigma,n}) \qquad (n = 1, 2, \ldots).$$

Theorem. *In order that a countable Abelian group without torsion $\mathfrak{G}$ could be decomposed into a direct sum of groups of the first rank it is necessary and sufficient that:* 1) *$\mathfrak{G}$ admits a regular representation and* 2) *for any σ and n such that $G_{\sigma,n} \overline{\in} \mathfrak{G}_{\sigma,n-1}$ the condition $[G_{\sigma,n}, \mathfrak{G}_{\sigma,n-1}]$ is satisfied.*

Proof. Necessity. If $\mathfrak{G}$ is decomposible into a direct sum of groups of the first rank, than all non-zero elements of all direct sum-

mands are regular and consequently $\mathfrak{G}$ admits a regular representation. Further, $\mathfrak{G}_{\sigma,n-1}$ is a direct summand of $\mathfrak{G}_{\sigma,n}$ (this can be proved with the help of theorem 2 of the paper «Some properties» etc.). If $G_{\sigma,n} \overline{\in} \mathfrak{G}_{\sigma,n-1}$, then the truth of the condition $[G_{\sigma,n}, \mathfrak{G}_{\sigma,n-1}]$ follows from this fact by theorem 1 of the paper referred to above.

Sufficiency. Enumerate all elements of $\mathfrak{G}$: $G_0 = 0, G_1, G_2, \ldots$ According to lemma 2, we can construct a sequence of finite sets Ω_0, $\Omega_1, \Omega_2, \ldots$ of subgroups of $\mathfrak{G}$ such that: 1) all the set Ω_n satisfy the condition [*]; 2) $\Omega_n \subset \Omega_{n+1}$; 3) $G_n \in \mathfrak{G}[\Omega_n]$. It is easily seen that $\mathfrak{G}$ is the direct sum of groups contained in the sum of Ω_n, and since these groups are themselves direct sums of groups of the first rank, everything is proved.

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MATHÉMATIQUES

SUR LE NOMBRE DES ÉLÉMENTS D'UN GROUPE DONT UNE PUISSANCE APPARTIENT À UN ENSEMBLE ARBITRAIRE D'ÉLÉMENTS

Par V. GROCHEFF

(Présenté par O. J. Schmidt, de l'Académie, le 8. V. 1939)

En 1925 M. Weissner dans son article «On the number of elements of a group which have a power in a given conjugate set» (1) a démontré le théorème suivant:

«Soit m l'ordre d'un élément dans une classe d'éléments conjugués $\mathfrak{M}$ d'un groupe $\mathfrak{G}$. Le nombre des éléments du groupe dont une certaine puissance appartient à la classe d'éléments conjugués $\mathfrak{M}$ est un multiple du plus grand diviseur de l'ordre du groupe qui est premier avec m». Le théorème cité peut être obtenu comme un cas particulier du théorème suivant très général établi en 1938 par M. Dubuque dans sa Note «Sur le théorème fondamental de Frobenius» (2).

«Soit m le plus petit multiple des ordres des éléments appartenant à un complexe invariant $\mathfrak{M}$ du groupe $\mathfrak{G}$. Soit n un diviseur de l'ordre du groupe $\mathfrak{G}$ et un multiple de m. Le nombre des éléments du groupe dont la n-ième puissance appartient à un complexe invariant $\mathfrak{A}$ arbitraire et dont une certaine puissance appartient au complexe invariant $\mathfrak{M}$ est un multiple du plus grand diviseur de n premier avec m».

En particulier, si dans le théorème que nous venons de citer on prend pour n l'ordre du groupe $\mathfrak{G}$ et si l'on considère l'unité comme un complexe invariant $\mathfrak{A}$, on obtient comme corollaire de ce théorème la généralisation suivante du théorème de Weissner:

«Soit m le plus petit multiple des ordres des éléments appartenant à un complexe invariant $\mathfrak{M}$ d'un groupe $\mathfrak{G}$. Le nombre des élements du groupe dont une certaine puissance appartient au complexe invariant $\mathfrak{M}$ est un multiple du plus grand diviseur de l'ordre du groupe premier avec m».

Le but de la présente Note est de démontrer une proposition qui généralise le théorème de M. Weissner ainsi que le corollaire cité du théorème de M. Dubuque. La méthode que nous suivrons dans notre démonstration a été développée par M. Dubuque d'une part dans la Note déjà citée et d'autre part dans sa Note «Un théorème contenant les théorèmes de Frobenius, de Weissner et de Turkin sur le nombre des éléments d'un ordre donné dans un groupe» (3).

Théorème 1. *Soit m le plus petit multiple des ordres des éléments appartenant à un système arbitraire $\mathfrak{M}$ d'éléments d'un groupe $\mathfrak{G}$ et $\mathfrak{N}$*

le normalisateur du systèm $\mathfrak{M}$. *Le nombre des éléments du groupe* $\mathfrak{G}$ *dont une certaine puissance appartient au système* $\mathfrak{M}$ *est un multiple du plus grand diviseur de l'ordre du normalisateur* $\mathfrak{N}$ *premier avec* m.

Démonstration. En vertu de l'énoncé du théorème, il faut déterminer le nombre des éléments X du groupe $\mathfrak{G}$ vérifiant la condition

$$X^{\alpha} \subseteq \mathfrak{M}, \tag{1}$$

α étant un nombre quelconque. Soit l le plus grand diviseur de l'ordre du groupe $\mathfrak{G}$ premier avec m.

Désignons X^l par Z

$$X^l = Z \tag{2}$$

et remplaçons la condition (1) par la relation

$$Z^{\beta} \subseteq \mathfrak{M}, \tag{3}$$

où β est un nombre quelconque.

Il est facile à voir que l'égalité (2) jointe à la relation (3) est équivalente à la condition (1). En effet, si un élément X_1 vérifie la condition (1) et si l'on détermine δ de la congruence

$$l\delta \equiv 1 \pmod{m},$$

on aura évidemment

$$X_1^{\alpha l\delta} \subseteq \mathfrak{M}.$$

En écrivant cette dernière relation dans la forme

$$Z_1 = X_1^l, \quad Z_1^{\alpha\delta} \subseteq \mathfrak{M}$$

nous en concluons que l'élément X_1 vérifie l'égalité (2) et l'égalité (3). Inversement, si un certain élément X vérifie l'égalité (2) et l'égalité (3), il en résulte que sa βl-ième puissance appartient au système $\mathfrak{M}$, c'est-à-dire l'élément considéré vérifie la condition (1) quand on prend α égal à βl.

Donc, au lieu de considérer la condition (1) nous allons considérer l'égalité (2) et la relation (3).

S'il existe un certain élément X_1 pour lequel les conditions posées sont vérifiées, nous devons avoir les égalités suivantes:

$$X_1^l = Z_1, \tag{4}$$

$$Z_1^{\beta} = M_1, \tag{5}$$

où l'élément M_1 appartient au système $\mathfrak{M}$. Soit n l'ordre du normalisateur $\mathfrak{N}$ du système $\mathfrak{M}$ et soit $\mathfrak{H}_1$ le groupe d'ordre h_1 qui est le normalisateur de l'élément Z_1. Soit ensuite $\mathfrak{N}_1$ le groupe d'ordre n_1 qui est la partie commune des groupes $\mathfrak{N}$ et $\mathfrak{H}_1$. Formons la suite complète des résidus

$$N_1 = 1, \quad N_2, \ldots, N_k$$

du groupe $\mathfrak{N}$ par rapport à son sous-groupe $\mathfrak{N}_1$. Tous les éléments

$$Z_i = N_i^{-1} Z_1 N_i \qquad (i = 1, 2, \ldots, k) \tag{6}$$

sont différents et vérifient la condition (3). Pour chaque élément du système (6) l'équation

$$X^l = Z_i$$

a le même nombre de solutions et ce nombre est un multiple du plus grand diviseur $D(l, h_1)$ des nombres l et h_1.

Le nombre total des éléments du groupe $\mathfrak{G}$ vérifiant l'une des conditions

$$X^l = Z_i \qquad (i = 1, 2, \ldots, k)$$

est donc un multiple du produit

$$kD(l, h_1)$$

et tous ces éléments vérifient l'équation (2) et la condition (3). Comme k est l'indice du sous-groupe $\mathfrak{N}_1$ par rapport au groupe $\mathfrak{N}$, on a

$$kn_1 = n. \tag{7}$$

Mais $\mathfrak{N}_1$ est aussi un sous-groupe du groupe $\mathfrak{H}_1$, donc n_1 est un diviseur de h_1, c'est pourquoi $D(l, h_1)$ est divisible par $D(l, n_1)$. Ainsi le nombre cherché des solutions est divisible par le produit

$$kD(l, n_1)$$

qui, d'ailleurs, en tenant compte de (7), est un multiple du plus grand diviseur $D(l, n)$ des nombres l et n. Nous en concluons que si pour un nombre donné β il existe un élément X_1 vérifiant les conditions (2) et (3), cet élément fait paraître dans le groupe $\mathfrak{G}$ un système de solutions des équations (2) et (3) pour la valeur donnée de β, et le nombre d'éléments de ce système est un multiple de $D(l, n)$. S'il existe un autre élément X_2 n'appartenant pas au système précédent et verifiant l'équation (3) et l'égalité complémentaire (2) pour la même valeur de β, on construit pour ce X_2 d'une manière analogue un système de solutions dont le nombre d'éléments est aussi un multiple de $D(l, n)$. En continuant ce procédé, nous obtenons enfin tous les éléments du groupe $\mathfrak{G}$ qui pour la valeur donnée de β vérifient les conditions (2) et (3), et d'ailleurs ce nombre sera encore un multiple de $D(l, n)$. Tous les raisonnements précédents sont vrais pour une valeur quelconque de β; donc, le nombre total des solutions vérifiant les conditions (2) et (3) pour toutes les valeurs possibles de β sera un multiple de $D(l, n)$, si toutefois ces solutions sont toutes différentes. Mais si deux solutions X et X' coïncident, alors $Z = X^l$ et $Z' = X'^l$ seront égaux et les systèmes de solutions construits pour ces éléments X et X' seront évidemment formés des mêmes éléments.

Faisons encore quelques remarques à propos du théorème intéressant suivant démontré dans la Note de M. Dubuque déjà citée [3].

«Soit a l'ordre d'un élément dans une classe $\mathfrak{A}$ d'éléments conjuguée du groupe $\mathfrak{G}$ et soit m l'ordre de tous les éléments dans un système arbitraire $\mathfrak{M}$ d'éléments du groupe $\mathfrak{G}$. Soit ensuite n un diviseur de l'ordre du groupe et un multiple de m. Si a et m sont premiers entre eux, le nombre des éléments du groupe $\mathfrak{G}$, dont la n-ième puissance appartient à la classe $\mathfrak{A}$ et dont une certaine puissance appartient au système $\mathfrak{M}$, est un multiple de $\varphi(m)$».

Tout d'abord dans le théorème cité il n'est pas nécessaire d'exiger que n soit un diviseur de l'ordre du groupe; il est probable que cette condition a été introduite dans l'énoncé du théorème pour cette seule raison qu'elle figure dans une proposition auxiliaire [4] utilisée par l'auteur dans la démonstration.

«Soient m et n des diviseurs de l'ordre du groupe et soit n un multiple de m. Le nombre des éléments du groupe dont l'ordre est un diviseur de n et dont une certaine puissance appartient à un système arbitraire d'éléments $\mathfrak{M}$ d'ordre m est un multiple de $\varphi(m)$».

Mais dans la démonstration de ce dernier théorème la condition précédente n'est pas du tout utilisée, donc ce théorème ainsi que celui qui

nous intéresse seront encore vrais quand on ne suppose plus que n est un diviseur de l'ordre du groupe.

La seconde remarque consiste en ce qu'au lieu d'une classe d'éléments conjugués on peut considérer un complexe invariant d'éléments du groupe. En effet, supposons qu'un complexe invariant $\mathfrak{A}$ est formé de k classes d'éléments conjugués $\mathfrak{A}_1, \mathfrak{A}_2, \ldots \mathfrak{A}_k$. Le nombre des solutions vérifiant les conditions

$$\left.\begin{aligned} X^n &\subseteq \mathfrak{A}_i, \\ X^\alpha &\subseteq \mathfrak{M}, \end{aligned}\right\} \qquad (1 \leqslant i \leqslant n) \tag{10}$$

α étant un nombre quelconque, est un multiple de $\varphi(m)$ en vertu du théorème de M. Dubuque. Le nombre total des solutions pour les $\mathfrak{A}_i$ différents $(1 \leqslant i \leqslant k)$ est encore un multiple de $\varphi(m)$. En effet, il ne peut évidemment exister de solutions qui vérifient en même temps deux ou plus de deux paires de conditions du type (10). En tenant compte de ces remarques on peut énoncer le théorème suivant:

Théorème 2. *Soit a le plus petit multiple des ordres des éléments d'un complexe $\mathfrak{A}$ d'un groupe $\mathfrak{G}$ et m l'ordre de tous les éléments dans un système arbitraire d'éléments $\mathfrak{M}$ du groupe $\mathfrak{G}$. Soit ensuite n un multiple de m. Si a et m sont premiers entre eux, le nombre des éléments du groupe $\mathfrak{G}$ dont la n-ième puissance appartient au complexe invariant $\mathfrak{A}$ et dont une certaine puissance appartient au système $\mathfrak{M}$ est un multiple de $\varphi(m)$.*

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MECHANICS

ON THE INSTABILITY OF VON KÁRMÁN'S VORTEX STREETS

By N. KOCHIN, Member of the Academy

As is known, von Kármán has given a necessary condition for the stability of a system consisting of two parallel chains of vortices with equal, but opposite circulations. According to this condition, each chain must be situated in such a way that its vortices which are supposed to be equidistant lie opposite the middle-points between the vortices of the other chain, and the distance l between two adjacent vortices of one chain and the distance h between the chains must be connected by the relation

$$\operatorname{sh}\frac{\pi h}{l}=1, \quad \text{or} \quad \operatorname{ch}\frac{\pi h}{l}=\sqrt{2}, \tag{1}$$

whence $h=0.2806\ldots l$.

However, this condition is only necessary for the stability. In fact, von Kármán obtained it by consideration of a special kind of displacements supposed to be infinitely small, so that in the equations of the disturbed motion of the vortices terms of orders of smallness higher than the first could be omitted.

We shall show that von Kármán's condition is not sufficient for the stability. Namely, we shall show that von Karmán's vortex streets are instable systems, if we consider the displacements of the vortices as arbitrarily small, but finite quantities and take the equations of the disturbed motion of the vortices in their complete form.

We make first the following preliminary remark. Consider two chains of vortices. It is said that the motion is stable, if after arbitrary (small) displacements of all or some of the vortices at an initial moment all vortices will remain at all subsequent times in the neighbourhood of the positions which they should have occupied were the motion to proceed undisturbed. Otherwise the motion is said to be instable. Under this general definition of stability the instability of von Kármán's vortex streets may be easily established without any calculations. Indeed, displace all vertices of one of the chains by one and the same small quantity in one and the same direction. Then all vortices both chains will proceed to move with one and the same velocity differing from the velocity of the undisturbed motion either in magnitude or in direction, and it becomes clear that all vortices will gradually move with small, but constant velocities away from the positions corresponding to the undisturbed state. The motion proves thus to be instable. It is, however, obvious that the configuration of the system will be preserved.

Therefore we change the definition of the stability of von Kármán's vortex streets in the following manner. We shall call a system of vortices stable, if for any arbitrarily small positive number ε such a positive number δ can be found that after any displacement of the vortices not exceeding δ the motion will proceed in such a way that the distance between any two vortices will always differ from their distance in the undisturbed state by no more than ε.

We now proceed to the proof of the instability of von Kármán's vortex streets. Suppose that in the plane of the complex variable $z=x+iy$ we have two chains of vortices. The one chain may consist of vortices of the intensity $+\Gamma$ situated at the points

$$\frac{l}{4}+\frac{ih}{2}+nl \qquad (n=0, \pm 1, \pm 2, \ldots),$$

and the other of vortices of the intensity $-\Gamma$ situated at the points

$$-\frac{l}{4}-\frac{ih}{2}+nl \qquad (n=0, \pm 1, \pm 2, \ldots).$$

We divide each of both chains into two, taking into the one all vortices with even n and into the other all vortices with uneven n. We thus obtain four chains of vortices, the representatives of which will be the four vortices situated at the points

$$z_{10}=\frac{l}{4}+\frac{ih}{2}, \quad z_{20}=\frac{5l}{4}+\frac{ih}{2}, \quad z_{30}=-\frac{l}{4}-\frac{ih}{2}, \quad z_{40}=\frac{3l}{4}-\frac{ih}{2}. \tag{2}$$

We give now to all vortices of each of the four chains one and the same displacements different for all four chains; the representatives of the four chains may now have the complex co-ordinates z_1, z_2, z_3, z_4.

The motion of the fluid caused by such a system of vortices possesses the complex velocity potential

$$w(z)=\frac{\Gamma}{2\pi i}\left\{\ln\sin\frac{\pi(z-z_1)}{2l}+\ln\sin\frac{\pi(z-z_2)}{2l}-\right.$$
$$\left.-\ln\sin\frac{\pi(z-z_3)}{2l}-\ln\sin\frac{\pi(z-z_4)}{2l}\right\}$$

and the complex velocity

$$\frac{dw}{dz}=\frac{\Gamma}{4li}\left\{\operatorname{ctg}\frac{\pi(z-z_1)}{2l}+\operatorname{ctg}\frac{\pi(z-z_2)}{2l}-\operatorname{ctg}\frac{\pi(z-z_3)}{2l}-\operatorname{ctg}\frac{\pi(z-z_4)}{2l}\right\}.$$

Hence for the complex velocity of the vertices themselves we obtain the following expressions:

$$\left.\begin{aligned}
\frac{d\bar{z}_1}{dt}&=\frac{\Gamma}{4li}\left\{\operatorname{ctg}\frac{\pi(z_1-z_2)}{2l}-\operatorname{ctg}\frac{\pi(z_1-z_3)}{2l}-\operatorname{ctg}\frac{\pi(z_1-z_4)}{2l}\right\},\\
\frac{d\bar{z}_2}{dt}&=\frac{\Gamma}{4li}\left\{\operatorname{ctg}\frac{\pi(z_2-z_1)}{2l}-\operatorname{ctg}\frac{\pi(z_2-z_3)}{2l}-\operatorname{ctg}\frac{\pi(z_2-z_4)}{2l}\right\},\\
\frac{d\bar{z}_3}{dt}&=\frac{\Gamma}{4li}\left\{\operatorname{ctg}\frac{\pi(z_3-z_1)}{2l}+\operatorname{ctg}\frac{\pi(z_3-z_2)}{2l}-\operatorname{ctg}\frac{\pi(z_3-z_4)}{2l}\right\},\\
\frac{d\bar{z}_4}{dt}&=\frac{\Gamma}{4li}\left\{\operatorname{ctg}\frac{\pi(z_4-z_1)}{2l}+\operatorname{ctg}\frac{\pi(z_4-z_2)}{2l}-\operatorname{ctg}\frac{\pi(z_4-z_3)}{2l}\right\}.
\end{aligned}\right\} \tag{3}$$

The integration of this system of equations gives us z_1, z_2, z_3, z_4 as functions of time, i. e. determines the motion of all four chains of vortices. In particular, if there were no initial disturbances, i. e. if the initial values of z_1, z_2, z_3, z_4 are given by the formulae (2), then all four chains will move paralle to the x-axis with one and the same velocity $\frac{\Gamma}{2l}\operatorname{th}\frac{\pi h}{l}$. We put therefore in the general case

$$z_k=\frac{\Gamma t}{2l}\operatorname{th}\frac{\pi h}{l}+z_{k0}+\frac{2l}{\pi}\zeta_k \qquad (k=1, 2, 3, 4), \tag{4}$$

where ζ_k are dimensionless displacements of the vortices with respect to the positions which they occupy in undisturbed motion. We further introduce the dimensionless time $\tau = \frac{\Gamma \pi t}{8l^2}$ and put $\frac{\pi h}{2l} = H$. The system (3) will then take the form

$$\left.\begin{aligned}
\frac{d\bar{\zeta}_1}{d\tau} &= i\left\{\operatorname{tg}(\zeta_1-\zeta_2)+\operatorname{ctg}\left(\frac{\pi}{4}+iH+\zeta_1-\zeta_3\right)+\right.\\
&\left.+\operatorname{ctg}\left(-\frac{\pi}{4}+iH+\zeta_1-\zeta_4\right)\right\}-2\operatorname{th}2H,\\
\frac{d\bar{\zeta}_2}{d\tau} &= i\left\{\operatorname{tg}(\zeta_2-\zeta_1)+\operatorname{ctg}\left(\frac{3\pi}{4}+iH+\zeta_2-\zeta_3\right)+\right.\\
&\left.+\operatorname{ctg}\left(\frac{\pi}{4}+iH+\zeta_2-\zeta_4\right)\right\}-2\operatorname{th}2H,\\
\frac{d\bar{\zeta}_3}{d\tau} &= i\left\{\operatorname{ctg}\left(\frac{\pi}{4}+iH+\zeta_1-\zeta_3\right)+\right.\\
&\left.+\operatorname{ctg}\left(\frac{3\pi}{4}+iH+\zeta_2-\zeta_3\right)-\operatorname{tg}(\zeta_3-\zeta_4)\right\}-2\operatorname{th}2H,\\
\frac{d\bar{\zeta}_4}{d\tau} &= i\left\{\operatorname{ctg}\left(-\frac{\pi}{4}+iH+\zeta_1-\zeta_4\right)+\right.\\
&\left.+\operatorname{ctg}\left(\frac{\pi}{4}+iH+\zeta_2-\zeta_4\right)-\operatorname{tg}(\zeta_4-\zeta_3)\right\}-2\operatorname{th}2H.
\end{aligned}\right\} \tag{5}$$

This system possesses the obvious integral

$$\zeta_1+\zeta_2-\zeta_3-\zeta_4 = \text{const.}$$

For the constant on the right hand side we take the value zero, so that

$$\zeta_4 = \zeta_1+\zeta_2-\zeta_3. \tag{6}$$

Under this assumption we shall obviously have

$$\frac{y_1+y_2}{2}-\frac{y_3+y_4}{2} = \frac{y_{10}+y_{20}}{2}-\frac{y_{30}+y_{40}}{2} = h$$

which shows that the mean distance between the vortices of the upper and the lower chains is the same as in the case of the undisturbed motion.

We introduce now the denotations

$$2(\zeta_3-\zeta_1) = \alpha, \quad 2(\zeta_4-\zeta_1) = \beta. \tag{7}$$

Using (6), we find from (5) after simple transformations the following system of equations for α and β:

$$\left.\begin{aligned}
\frac{d\bar{\alpha}}{d\tau} &= 4i\sin\beta\left(\frac{1}{\cos\alpha+\cos\beta}-\frac{1}{\cos\beta-i\operatorname{sh}2H}\right),\\
\frac{d\bar{\beta}}{d\tau} &= 4i\sin\alpha\left(\frac{1}{\cos\alpha+\cos\beta}-\frac{1}{\cos\alpha+i\operatorname{sh}2H}\right).
\end{aligned}\right\} \tag{8}$$

Clearly, the instability of von Kármán's vortex streets in the above sense will be established, if we prove that the solution $\alpha=0$, $\beta=0$ of the system (8) is instable in the sense of Liapunoff.

The system of the first approximation which is obtained, if in the right hand sides of the system (8) we retain only terms linear in α and β, has the form

$$\frac{d\bar{\alpha}}{d\tau} = -\frac{2i(1+i\operatorname{sh}2H)}{1-i\operatorname{sh}2H}\beta, \qquad \frac{d\bar{\beta}}{d\tau} = -\frac{2i(1-i\operatorname{sh}2H)}{1+i\operatorname{sh}2H}\alpha.$$

Adding two conjugated equations

$$\frac{d\alpha}{d\tau} = \frac{2i\,(1 - i\,\mathrm{sh}\,2H)}{1 + i\,\mathrm{sh}\,2H}\,\bar{\beta}, \qquad \frac{d\beta}{d\tau} = \frac{2i\,(1 + i\,\mathrm{sh}\,2H)}{1 - i\,\mathrm{sh}\,2H}\,\bar{\alpha},$$

we see at once that the roots $\varkappa$ of the determining equation have the form

$$\varkappa = \pm 2\left\{\frac{1 - \mathrm{sh}^2 2H}{\mathrm{ch}^2 2H} \pm 2i\,\frac{\mathrm{sh}\,2H}{\mathrm{ch}^2 2H}\right\}.$$

If $\mathrm{sh}\,2H$ is different from 1, two of the roots of the determining equation will have a positive real part and, consequently, the solution $\alpha = 0$, $\beta = 0$ will be an instable solution of the system (8). It remains to consider the case investigated by von Kármán, when

$$\mathrm{sh}\,2H = \mathrm{sh}\,\frac{\pi h}{l} = 1, \quad \mathrm{ch}\,2H = \sqrt{2}.$$

The system (8) takes in this case the form

$$\left.\begin{aligned} \frac{d\bar{\alpha}}{d\tau} &= 4i\sin\beta\left(\frac{1}{\cos\alpha + \cos\beta} - \frac{1}{\cos\beta - i}\right), \\ \frac{d\bar{\beta}}{d\tau} &= 4i\sin\alpha\left(\frac{1}{\cos\alpha + \cos\beta} - \frac{1}{\cos\alpha + i}\right). \end{aligned}\right\} \tag{9}$$

Since the determining equation of the system of the first approximation has in this case two purely imaginary roots $\pm 2i$ of multiplicity 2, we have before us the case, when the first approximation is insufficient for the investigation of the stability of the solution $\alpha = 0$, $\beta = 0$. Put

$$\alpha = \alpha_1 + i\alpha_2, \qquad \beta = \beta_1 + i\beta_2, \qquad F = 4\ln\left|\frac{(\cos\alpha + i)\,(\cos\beta - i)}{\cos\alpha + \cos\beta}\right|. \tag{10}$$

The system (9) may be then written in the form

$$\frac{d\alpha_1}{d\tau} = \frac{\partial F}{\partial\beta_2}, \quad \frac{d\alpha_2}{d\tau} = -\frac{\partial F}{\partial\beta_1}, \quad \frac{d\beta_1}{d\tau} = \frac{\partial F}{\partial\alpha_2}, \quad \frac{\partial\beta_2}{\partial\tau} = -\frac{\partial F}{\partial\alpha_1}. \tag{11}$$

This system has the obvious integral

$$F = \mathrm{const.} \tag{12}$$

Let the initial conditions be such that the constant takes the value zero, so that we have the integral

$$\left.\begin{aligned} F = &-2\alpha_1\alpha_2 + 2\beta_1\beta_2 + \tfrac{1}{8}\alpha_1^4 - \tfrac{2}{3}\alpha_1^3\alpha_2 - \tfrac{3}{4}\alpha_1^2\alpha_2^2 + \\ &+ \tfrac{2}{3}\alpha_1\alpha_2^3 + \tfrac{1}{8}\alpha_2^4 + \tfrac{1}{4}(\alpha_1^2 - \alpha_2^2)(\beta_1^2 - \beta_2^2) - \alpha_1\alpha_2\beta_1\beta_2 + \\ &+ \tfrac{1}{8}\beta_1^4 + \tfrac{2}{3}\beta_1^3\beta_2 - \tfrac{3}{4}\beta_1^2\beta_2^2 - \tfrac{2}{3}\beta_1\beta_2^3 + \tfrac{1}{8}\beta_2^4 + \ldots = 0. \end{aligned}\right\} \tag{13}$$

The system of equations (11) takes the form

$$\left.\begin{aligned} \frac{d\alpha_1}{d\tau} &= 2\beta_1 - \tfrac{1}{2}\beta_2(\alpha_1^2 - \alpha_2^2) - \alpha_1\alpha_2\beta_1 + \\ &+ \tfrac{2}{3}\beta_1^3 - \tfrac{3}{2}\beta_1^2\beta_2 - 2\beta_1\beta_2^2 + \tfrac{1}{2}\beta_2^3 + \ldots, \\ \frac{d\alpha_2}{d\tau} &= -2\beta_2 - \tfrac{1}{2}\beta_1(\alpha_1^2 - \alpha_2^2) + \alpha_1\alpha_2\beta_2 - \\ &- \tfrac{1}{2}\beta_1^3 - 2\beta_1^2\beta_2 + \tfrac{3}{2}\beta_1\beta_2^2 + \tfrac{2}{3}\beta_2^3 + \ldots, \\ \frac{d\beta_1}{d\tau} &= -2\alpha_1 - \tfrac{2}{3}\alpha_1^3 - \tfrac{3}{2}\alpha_1^2\alpha_2 + 2\alpha_1\alpha_2^2 + \\ &+ \tfrac{1}{2}\alpha_2^3 - \tfrac{1}{2}\alpha_2(\beta_1^2 - \beta_2^2) - \alpha_1\beta_1\beta_2 + \ldots, \\ \frac{d\beta_2}{d\tau} &= 2\alpha_2 - \tfrac{1}{2}\alpha_1^3 + 2\alpha_1^2\alpha_2 + \tfrac{3}{2}\alpha_1\alpha_2^2 - \\ &- \tfrac{2}{3}\alpha_2^3 - \tfrac{1}{2}\alpha_1(\beta_1^2 - \beta_2^2) + \alpha_2\beta_1\beta_2 + \ldots, \end{aligned}\right\} \tag{14}$$

where the omitted terms are of the fifth and higher powers in α_1, α_2, β_1, β_2.

Applying the methods of A. Liapunoff, we find that on ground of the preceding equations we have

$$\left.\begin{aligned}\frac{d(\alpha_1^2+\beta_1^2-\alpha_2^2-\beta_2^2+G_1)}{d\tau} &= -(\alpha_1^2+\beta_1^2+\alpha_2^2+\beta_2^2)(\alpha_1\beta_2+\alpha_2\beta_1)+\ldots,\\ \frac{d(\alpha_1\beta_2+\alpha_2\beta_1+G_2)}{d\tau} &= -\frac{1}{2}(\alpha_1^2+\beta_1^2)^2+\frac{1}{2}(\alpha_2^2+\beta_2^2)^2+\\ &+(\alpha_1\alpha_2-\beta_1\beta_2)(\alpha_1^2+\beta_1^2+\alpha_2^2+\beta_2^2)+\ldots,\end{aligned}\right\} \quad (15)$$

where the omitted terms are of the sixth and higher powers in α_1, α_2, β_1, β_2 and

$$\left.\begin{aligned}G_1 &= \frac{1}{24}(\alpha_1^4-6\alpha_1^2\beta_1^2+\beta_1^4)-\frac{1}{4}(\alpha_1^2-\beta_1^2)(\alpha_2^2-\beta_2^2)-\alpha_1\alpha_2\beta_1\beta_2+\\ &+\frac{1}{24}(\alpha_2^4-6\alpha_2^2\beta_2^2+\beta_2^4)-\frac{1}{8}\alpha_1\alpha_2(\alpha_2^2-3\beta_2^2-\alpha_1^2+3\beta_1^2)-\\ &-\frac{1}{8}\beta_1\beta_2(3\alpha_2^2-\beta_2^2-3\alpha_1^2+\beta_1^2),\\ G_2 &= \frac{1}{24}\{\beta_1\alpha_2(3\alpha_1^2-\beta_1^2+\alpha_2^2-3\beta_2^2)-\alpha_1\beta_2(\alpha_1^2-3\beta_1^2+3\alpha_2^2-\beta_2^2)\}.\end{aligned}\right\} \quad (16)$$

Taking into account the condition (13), we may also write

$$\frac{d(\alpha_1\beta_2+\alpha_2\beta_1+G_2)}{d\tau} = -\frac{1}{2}(\alpha_1^2+\beta_1^2+\alpha_2^2+\beta_2^2)(\alpha_1^2+\beta_1^2-\alpha_2^2-\beta_2^2)+\ldots. \quad (17)$$

Introducing the function

$$V=(\alpha_1^2+\beta_1^2-\alpha_2^2-\beta_2^2+G_1)(\alpha_1\beta_2+\alpha_2\beta_1+G_2), \quad (18)$$

we obtain in virtue of the preceding equations and the equation (13):

$$\frac{dV}{d\tau} = -\frac{1}{2}(\alpha_1^2+\beta_1^2+\alpha_2^2+\beta_2^2)[(\alpha_1^2+\beta_1^2)^2+(\alpha_2^2+\beta_2^2)^2]+\ldots, \quad (19)$$

where the omitted terms are of the eighth and higher powers.

By an adequate choice of the variables α_1, α_2, β_1, β_2, arbitrarily small in modulus and satisfying the equation (13), the function V may be made negative. Since in virtue of (19) the derivative of V is negative for all α_1, α_2, β_1, β_2, sufficiently small in modulus, but not simultaneously equal to zero, from one of the theorems of A. Liapunoff it follows that the solution $\alpha=0$, $\beta=0$ of the system (9) is an instable solution and our theorem is proved.

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THEORY OF ELASTICITY

ON SAINT-VENANT'S PROBLEM FOR NATURALLY TWISTED BARS. I

By A. LOURIE and G. JANELIDZE

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In the present paper we deduce the equations of the theory of elasticity in a form convenient for the solution of problems connected with the consideration of «naturally twisted» bars. Thus, we call a bar bound by a surface arising at the motion of a plane figure rotating with a constant angular velocity r, while its center of gravity moves along the axis perpendicular to the plane of the figure with the velocity *1*. In the plane of the figure we introduce the system of co-ordinates x_1, x_2, having their origin in the center of gravity of the figure and directed along the principal axes of inertia. The fixed cartesian axes x, y, z are chosen in such a way that in the initial position of the figure x_1 and x_2 coincide with x and y, and the axis z (being at the same time the axis x_3) is perpendicular to the plane of the figure. The three numbers x_1, x_2, x_3 represent curvilinear co-ordinates connected with the cartesian co-ordinates by the relations

$$x_1 = x\cos rz + y\sin rz, \quad x_2 = -x\sin rz + y\cos rz, \quad x_3 = z \tag{1}$$

and, conversely,

$$x = x_1\cos rx_3 - x_2\sin rx_3, \quad y = x_1\sin rx_3 + x_2\cos rx_3, \quad z = x_3. \tag{2}$$

We obtain the equation of the lateral surface of the bar, putting in (2)

$$x_1 = x_1(s), \quad x_2 = x_2(s), \tag{3}$$

where the equations (3) represent the parametric equations of the contour (s is the length of the arc of the contour).

In difference from the usual denotations of the tensor analysis we understand under x_1, x_2, x_3 the contravariant components of the radius vector. With this exception we adopt the usual denotations of the tensor analysis.

The fundamental expressions relating to the (non-orthogonal) curvilinear system are:

a) The covariant components of the metrical tensor:

$$g_{11} = 1, \quad g_{12} = 0, \quad g_{13} = -x_2 r,$$
$$g_{22} = 1, \quad g_{23} = x_1 r,$$
$$g_{33} = 1 + (x_1^2 + x_2^2)\, r^2.$$

b) Its contravariant components:

$$g^{11}=1+x_2^2r^2,\quad g^{12}=-x_1x_2r^2,\quad g^{13}=x_2r,$$
$$g^{22}=1+x_1^2r^2,\quad g^{23}=-x_1r,$$
$$g^{33}=1.$$

c) The non-vanishing first and second Cristoffel's symbols:

$$[33,1]=-x_1r^2,\quad [33,2]=-x_2r^2,\quad [23,1]=-r,\quad [13,2]=r,$$
$$[13,3]=x_1r^2,\quad [23,3]=x_2r^2,$$
$$\left\{\begin{matrix}1\\33\end{matrix}\right\}=-x_1r^2,\quad \left\{\begin{matrix}2\\33\end{matrix}\right\}=-x_1r^2,\quad \left\{\begin{matrix}2\\13\end{matrix}\right\}=r,\quad \left\{\begin{matrix}1\\23\end{matrix}\right\}=-r.$$

As is known (1), the equilibrium equations of the continuous medium, volume forces being absent, have the form: in the interior (in absence of volume forces)

$$\frac{\partial\sigma^{rs}}{\partial x_s}+\sigma^{mn}\left\{\begin{matrix}r\\mn\end{matrix}\right\}=0,\quad \sigma^{rs}=\sigma^{sr},\tag{4}$$

on the lateral (free) surface,

$$\sigma^{rs}n_s=0.\tag{5}$$

Here and in what follows σ^{rs} denote the contravariant components of the stress tensor and n_s — the covariant components of the normal to the lateral surface. Calculating them, we find

$$n_1=\frac{1}{\delta}\nu_1,\quad n_2=\frac{1}{\delta}\nu_2,\quad n_3=0,\tag{6}$$

where $\delta^2=1+r^2\left(x_1\frac{dx_1}{ds}+x_2\frac{dx_2}{ds}\right)^2$, while

$$\nu_1=\frac{dx_2}{ds},\quad \nu_2=-\frac{dx_1}{ds}\tag{7}$$

represent the projections of the normal to the contour of the figure on the axes x_1, x_2. In virtue of this the equation (5) takes the form

$$\left.\begin{aligned}&\sigma^{11}\nu_1+\sigma^{12}\nu_2=0,\\&\sigma^{21}\nu_1+\sigma^{22}\nu_2=0,\\&\sigma^{31}\nu_1+\sigma^{32}\nu_2=0,\end{aligned}\right\}\tag{8}$$

and the equations (4) in our case will be:

$$\left.\begin{aligned}&\frac{\partial\sigma^{11}}{\partial x_1}+\frac{\partial\sigma^{12}}{\partial x_2}+\frac{\partial\sigma^{13}}{\partial x_3}-x_1r^2\sigma^{33}-2\sigma^{23}r=0,\\&\frac{\partial\sigma^{21}}{\partial x_1}+\frac{\partial\sigma^{22}}{\partial x_2}+\frac{\partial\sigma^{23}}{\partial x_3}-x_2r^2\sigma^{33}+2\sigma^{13}r=0,\\&\frac{\partial\sigma^{31}}{\partial x_1}+\frac{\partial\sigma^{32}}{\partial x_2}+\frac{\partial\sigma^{33}}{\partial x_3}=0.\end{aligned}\right\}\tag{9}$$

The equations expressing the generalized Hook's law in an arbitrary system of co-ordinates have the form (1):

$$\sigma^{rs}=\lambda\vartheta g^{rs}+\mu\left(g^{sk}\frac{\partial u^r}{\partial x_k}+g^{rk}\frac{\partial u^s}{\partial x_k}-\frac{\partial g^{rs}}{\partial x_k}u^k\right),\tag{10}$$

where u^k are the contravariant components of the displacement vector, λ and μ are Lamé's constants,

$$\vartheta=\frac{\partial u^r}{\partial x_r}+\left\{\begin{matrix}r\\rt\end{matrix}\right\}u^t.$$

In our case the equations (10) are as follows:

$$
\left.
\begin{aligned}
\sigma^{11} &= \lambda\vartheta\,(1+x_2^2r^2)+2\mu\left[\,(1+x_2^2r^2)\frac{\partial u^1}{\partial x_1}-x_1x_2r^2\frac{\partial u^1}{\partial x_2}+x_2r\frac{\partial u^1}{\partial x_3}-x_2r^2u^2\,\right],\\
\sigma^{22} &= \lambda\vartheta\,(1+x_1^2r^2)+2\mu\left[\,-x_1x_2r^2\frac{\partial u^2}{\partial x_1}+(1+x_1^2r^2)\frac{\partial u^2}{\partial x_2}-\right.\\
&\qquad\left.-x_1r\frac{\partial u^2}{\partial x_3}-x_1r^2u^1\,\right],\\
\sigma^{33} &= \lambda\vartheta+2\mu\left(x_2r\frac{\partial u^3}{\partial x_1}-x_1r\frac{\partial u^3}{\partial x_2}+\frac{\partial u^3}{\partial x_3}\right),\\
\sigma^{12} &= -\lambda\vartheta x_1x_2r^2+\mu\left[\,-x_1x_2r^2\frac{\partial u^1}{\partial x_2}+(1+x_1^2r^2)\frac{\partial u^1}{\partial x_2}-x_1r\frac{\partial u^1}{\partial x_3}+\right.\\
&\qquad\left.+(1+x_2^2r^2)\frac{\partial u^2}{\partial x_1}-x_1x_2r^2\frac{\partial u^2}{\partial x_2}+x_2r\frac{\partial u^2}{\partial x_3}+x_2r^2u^1+x_1r^2u^2\,\right],\\
\sigma^{13} &= \lambda\vartheta x_2r+\mu\left[\,x_2r\frac{\partial u^1}{\partial x_1}-x_1r\frac{\partial u^1}{\partial x_2}+\frac{\partial u^1}{\partial x_3}+(1+x_2^2r^2)\frac{\partial u^3}{\partial x_1}-\right.\\
&\qquad\left.-x_1x_2r^2\frac{\partial u^3}{\partial x_2}+x_2r\frac{\partial u^3}{\partial x_3}+ru^2\,\right],\\
\sigma^{23} &= -\lambda\vartheta x_1r+\mu\left[\,x_2r\frac{\partial u^2}{\partial x_1}-x_1r\frac{\partial u^2}{\partial x_2}+\frac{\partial u^2}{\partial x_3}-\right.\\
&\qquad\left.-x_1x_2r^2\frac{\partial u^3}{\partial x_1}+(1+x_1^2r^2)\frac{\partial u^3}{\partial x_2}-x_1r\frac{\partial u^3}{\partial x_3}+ru^1\,\right],\\
\vartheta &= \frac{\partial u^1}{\partial x_1}+\frac{\partial u^2}{\partial x_2}+\frac{\partial u^3}{\partial x_3}.
\end{aligned}
\right\}\qquad(11)
$$

The aggregate of stresses acting in the terminal cross-section of the bar is statically equivalent to the stress resultant with the components X, Y, Z along the cartesian axes and the components P, Q, R along the axes x_1, x_2, x_3 in the cross-section $x_3=l$ (l is the length of the bar) and to the stress couple, the components of which are respectively m_x, m_y, m_z, L, M, N. Forming the usual equilibrium conditions in the terminal cross-section expressed through X, Y, Z, m_x, m_y, m_z, and the components of the stress tensor in the cartesian system, and passing to P, Q, R, L, M, N and the contravariant components of the stress tensor, we obtain

$$
\left.
\begin{aligned}
&\iint\sigma^{31}do=P+rL,\qquad \iint\sigma^{32}do=Q+rM,\qquad \iint\sigma^{33}do=R,\\
&\iint x_2\sigma^{33}do=L,\qquad -\iint x_1\sigma^{33}do=M,\\
&\iint\left[(x_1\sigma^{32}-x_2\sigma^{31})+r\,(x_1^2+x_2^2)\,\sigma^{33}\right]do=N.
\end{aligned}
\right\}\qquad(12)
$$

For the solution of the obtained system of equations (9), (8), (11), (12) we put

$$
\sigma^{ik}=\sum_{\nu=0}^{\infty}r^\nu\sigma_\nu^{ik},\qquad u^k=\sum_{\nu=0}^{\infty}r^\nu u_\nu^k.\qquad(13)
$$

We assume that the «torsion» r is so small that these series converge. As the zero approximation we find the system of equations of the theory of elasticity in cartesian co-ordinates.

The equations of the first approximation have the form:

$$\left.\begin{aligned} \frac{\partial \sigma_1^{11}}{\partial x_1}+\frac{\partial \sigma_1^{12}}{\partial x_2}+\frac{\partial \sigma_1^{13}}{\partial x_3}-2\sigma_0^{23}=0, \\ \frac{\partial \sigma_1^{21}}{\partial x_1}+\frac{\partial \sigma_1^{22}}{\partial x_2}+\frac{\partial \sigma_1^{23}}{\partial x_3}+2\sigma_0^{13}=0, \quad \frac{\partial \sigma_1^{31}}{\partial x_1}+\frac{\partial \sigma_1^{32}}{\partial x_2}+\frac{\partial \sigma_1^{33}}{\partial x_3}=0, \end{aligned}\right\} \tag{14}$$

$$\sigma_1^{11}\nu_1+\sigma_1^{12}\nu_2=0, \quad \sigma_1^{21}\nu_1+\sigma_1^{22}\nu_2=0, \quad \sigma_1^{31}\nu_1+\sigma_1^{32}\nu_2=0, \tag{15}$$

$$\left.\begin{aligned} \iint \sigma_1^{31}do=L, \quad \iint \sigma_1^{32}do=M, \quad \iint \sigma_1^{33}do=0, \quad \iint \sigma_1^{33}x_2do=0, \\ \iint \sigma_1^{33}x_1do=0, \\ \iint (x_1\sigma_1^{32}-x_2\sigma_1^{31})\,do=-\iint (x_1^2+x_2^2)\,\sigma_0^{33}do. \end{aligned}\right\} \tag{16}$$

$$\left.\begin{aligned} \sigma_1^{11}&=\lambda\vartheta_1+2\mu\left(\frac{\partial u_1^1}{\partial x_1}+x_2\frac{\partial u_0^1}{\partial x_3}\right), \\ \sigma_1^{22}&=\lambda\vartheta_1+2\mu\left(\frac{\partial u_1^2}{\partial x_2}-x_1\frac{\partial u_0^2}{\partial x_3}\right), \\ \sigma_1^{33}&=\lambda\vartheta_1+2\mu\left(\frac{\partial u_1^3}{\partial x_3}+x_2\frac{\partial u_0^3}{\partial x_1}-x_1\frac{\partial u_0^3}{\partial x_2}\right), \\ \sigma_1^{12}&=\mu\left(\frac{\partial u_1^1}{\partial x_2}+\frac{\partial u_1^2}{\partial x_1}+x_2\frac{\partial u_0^2}{\partial x_3}-x_1\frac{\partial u_0^1}{\partial x_3}\right), \\ \sigma_1^{13}&=\lambda\vartheta_0x_2+\mu\left(\frac{\partial u_1^1}{\partial x_3}+\frac{\partial u_1^3}{\partial x_1}+x_2\frac{\partial u_0^1}{\partial x_1}-x_1\frac{\partial u_0^1}{\partial x_2}+x_2\frac{\partial u_0^3}{\partial x_3}-u_0^1\right), \\ \sigma_1^{23}&=-\lambda\vartheta_0x_1+\mu\left(\frac{\partial u_1^2}{\partial x_3}+\frac{\partial u_1^3}{\partial x_2}-x_1\frac{\partial u_0^2}{\partial x_2}+x_2\frac{\partial u_0^2}{\partial x_1}-x_1\frac{\partial u_0^3}{\partial x_3}+u_0^1\right), \\ \vartheta_1&=\frac{\partial u_1^1}{\partial x_1}+\frac{\partial u_1^2}{\partial x_2}+\frac{\partial u_1^3}{\partial x_3}. \end{aligned}\right\} \tag{17}$$

The applications of these equations to the Saint-Venant's problem will be given in the following communications.

Leningrad Industrial Institute.

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PHYSICS

MAGNETIC PROPERTIES OF THIN SUPERCONDUCTIVE FILMS

By N. E. ALEKSEEVSKY

(Communicated by P. L. Kapitza, Member of the Academy, 19. IV. 1939)

Research previously accomplished with thin superconductive films (1) has shown that in these films resistance appears with external fields that are larger than is the case for massive metals. Along with this, the superconductivity of these films is liable to be destroyed by a very weak current, whose field is considerably smaller than the critical field of a massive metal. Moreover, Misener, while investigating the penetration of the field into a cylinder limited by a thin film (2). discovered that the value of the external field with which the penetration of the field inside the cylinder takes place is also smaller than the critical field for a massive superconductor. Thus, the films had three critical values of the magnetic field and on account of this greatly resembled superconductive alloys. Naturally enough, it seemed desirable to explan the anomalies of thin superconductive films by the fact that they were not sufficiently pure, and therefore virtually were alloys (all the abovementioned films were obtained by electroplating or tinning). Another fact which pointed to the contamination of the films obtained by this method was a certain critical thickness of film reported in the aforesaid works (i. e. that with films of a smaller thickness, superconductivity failed to appear). On the other hand, the experiments of A. I. Shalnikov (3) who obtained thin films by means of the volatization of metal in vacuum while working in an apparatus held at the temperature of liquid helium, showed that the films remain superconductive up to a thickness$\approx 10^{-7}$ cm. Since in this case superconductivity was ascertained only through disappearance of resistance, there was not yet reason enough to uphold with confidence the influence of the dimensions of the superconductor upon the magnitude of the critical field. As the most characteristic property of alloys is the absence of abrupt change in induction per salta when superconductivity is destroyed by a field (4), it appeared of interest to ascertain the dependence of induction or magnetic moment upon external field for thin films. In this case, if the change in the magnetic moment had begun with fields smaller than the critical field for a pure metal, it would be natural to explain the anomalies of the films by the contaminations which might possibly occur on account of the great ratio of the surface to volume. Conversely, should the change of the magnetic moment take place with fields greater than the critical field for a pure metal, and standing close to those with which resistance makes its appearance, it would be necessary to seek for an explanation of the

anomalies in the small thickness of the films. It was to clear up this circumstance that the present work was undertaken.

In this work determination is given to that value of the external field at which a change in the magnetic moment of a tin film takes place, this change corresponding to the transition of the film from a superconductive to a normal state. The films were obtained by means of the evaporation of tin in vacuum, the apparatus being at the temperature of liquid helium. This afforded the possibility to obtain sufficiently pure films, in fact almost free from contamination by gas, since the vapour density of air at the temperature of liquid helium is insignificant*.

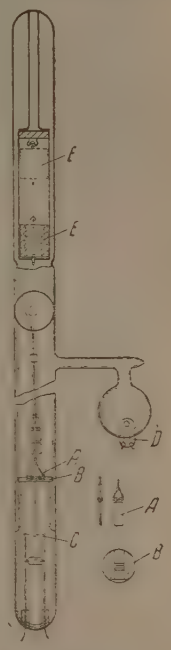

Fig. 1.

The thickness of the films was determined on the basis of data for the weight of the evaporated metal, as well as the magnitude of the radius of the sphere on which the lining plate was set. The lining plate was made of glass or quartz and was supplied with a glass or quartz hinged joint respectively. A special apparatus from which air was preliminarily pumped out to set up a high vacuum was used for measurements. The lay-out of the apparatus is schematically represented in Fig. 1.

The experiment was conducted in the following way: when the suspension system was let down, and the lining plate *A* was made to lie on diaphragm *B*, the Dewar vessel was filled with helium. Next, the volatization of the metal from discharger *C* was effected and the apparatus was placed between the two poles of a magnet. Now, with the help of a coil mounted upon the head of the apparatus, iron armature *E* and the suspension system thereto attached were lifted, whereupon helium was let out of container *D* in such an amount as was necessary for establishing thermal contact between the liquid helium in the Dewar vessel and the lining plate. The magnet was next placed in such a way that the plane of the plate might form a small angle with the field. On supplying current to the magnet, the plate would take a position conforming to the field (for this it was, of course, necessary that the suspension should be sufficiently thin). With further increase of the magnetic field, the plate would at a certain moment return back again to its initial position. This value of the field was precisely the critical one. The curve of the dependence of deviations upon external field is represented in Fig. 2.

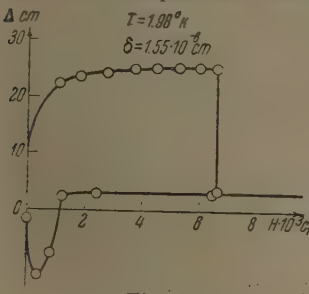

Fig. 2.

From this curve it will be seen that the film shows very considerable hysteresis. In order to get rid of the considerable residual moment set up by this hysteresis, the superconductivity of the film was destroyed before each lowering of temperature. For this purpose a beam of light emitted by a voltaic arc and running through a slit of the Dewar vessel was focussed onto the film. This permitted the temperature of the film to be raised above the critical point.

All the films measured in the present work were broken down by a

* Evaporation in an apparatus held at the temperature of liquid helium was for the first time tried by A. I. Shalnikov [3].

longitudinal magnetic field considerably exceeding the critical fields of massive tin. The curves representing the dependence of the critical field upon temperature obtained on a film of $7.24 \cdot 10^{-6}$ cm in thickness are to be found in Fig. 3.

Curve *1* refers to a freshly deposited film and curve *2*—to a film heated up to room temperature and then measured again*.

In this case, just as in the above cited work of A. Shalnikov, freshly deposited films are found to possess higher critical fields than films heated up to room temperature. Unfortunately, it proved impossible to detect the rise in critical temperature (kT) observed by A. I. Shalnikov ([3]) in freshly deposited films, as the degree of accuracy of our measurement fell markedly with an approach to kT**.

A decline in the precision of the measurements with rise in temperature

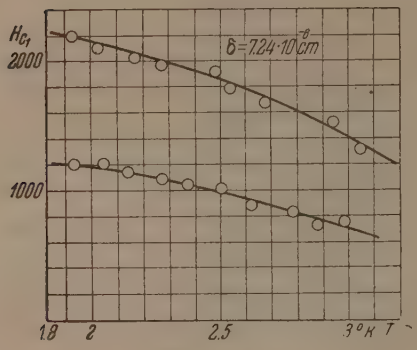

Fig. 3.

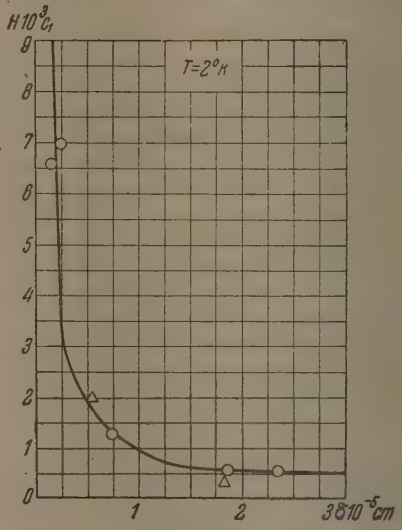

Fig. 4.

resulted possibly from insreasing the depth to which the field penetrated which caused the reduction of the magnetic moment per unit volume of the film. Such a change in the depth of penetration of the field was recently observed by Appleyard, Brestow, London ([5]) on mercury films and also by Shoenberg ([6]) on the colloids of mercury.

If we take the critical value of the field for films of different thickness at a sufficiently low temperature, it will be possible to obtain a clear idea of the character of the dependence of the critical field upon film thickness***.

In Fig. 4 are to be found curves of the dependence of the critical field upon the thickness of the film for 2°. The plotted curve refers to films heated up to room temperature. Two points marked △ correspond to the fields with which resistance is liable to appear. These data were kindly communicated by A. I. Shalnikov to whom the author expresses his ack-

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* Notice should be taken of the fact that the repeated heatings up to room temperature no longer produced any change in the critical value of the field.

** It is to be noted that the conditions of deposition of films as available in the present work fail to be wholly equivalent to the respective conditions stated in the work of A. I. Shalnikov, since in the present case the lining plate had no direct contact with liquid helium and could, therefore, become slightly heated.

*** The presence of hysteresis may be interpreted as a consequence of difference in the thickness of the film. Therefore in order to obtain an exact curve characteristic of the dependence of the critical field upon thickness, it would have been necessary to measure a considerable number of films of different thickness.

nowledgment. The comparatively good coincidence of these points with the curve is indicative of the purity of the films obtained and disproves the formerly maintained analogy between the properties of thin films and alloys.

In the light of the high values of the field at which a change in the magnetic moment of the film occurs, we may consider as highly plausible that an alloyshould be conceived as, a system of thin superconductive paths formed in consequence of the local heterogeneity of concentration in the given material (7).

From the curve it will be seen that up to thicknesses of $\simeq 10^{-5}$ cm the critical field is liable to grow rather slowly, increasing approximately 2—3 fold as compared with the critical field of massive tin. With thicknesses under 10^{-5} cm there begins a sharp rise of the critical field; and with films whose thickness amounts to nearly 10^{-6} cm, the critical field exceeds about 30 times the critical field of massive tin.

On the ground of the aforesaid the conclusion can be made that even the thinnest films are endowed with an anomalous magnetic moment, and consequently the depth of penetration of a field is with these films less than their thicknesses. Seeing that in the work already mentioned (5) it was ascertained that the depth of penetration is independent of thickness for films ot 10^{-5} cm, it will be safe to suppose that the depth of penetration of a field begins to depend upon thickness only after the latter is below 10^{-5} cm. In order to corroborate these assumptions it will be necessary to carry out quantitative measurements of magnetic moment per unit volume as a function of thickness. These measurements will be a sequel to the present work.

The present work was accomplished at the Institute of Physical Problems of the Academy of Sciences of the USSR.

Ukrainian
Physico-Technical Institute.

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PHYSIQUE

RADIATION DIFFUSE DES NEUTRONS PAR LE CYCLOTRON

Par D. ALKHASOV, I. KURTCHATOV, M. MESTCHERJAKOV et A. V. ROUKAVICHNIKOV

(Présenté par V. G. Khlopin, de l'Académie, le 4. V. 1939)

En travaillant avec un cyclotron à chambre d'accélération de type ordinaire, nous avons trouvé que le maximum de la radiation des neutrons avait lieu à une pression élevée de deutérium dans la chambre et non pas dans le cas où les deutons rapides tombaient sur la cible de lithium. L'élévation de la pression dans la chambre diminuait le flux de deutons dirigé vers la cible de la valeur d'un μA jusqu'à la disparition pratiquement complète et en même temps augmentait le rendement total de neutrons.

Nous avons expérimenté avec une chambre d'accélération où le rayon du dernier parcours du deuton était de 18 cm. Dans ces premières expériences le champ magnétique était maintenu à la valeur de 16 000 Oe, ce qui offrait la possibilité d'obtenir des deutons à l'énergie de 2 millions eV. La chambre travaillait dans le régime de décharge de deutérium pure dont la pression était 10^{-3}—10^{-4} mm Hg, et la HF-tension était 20—30 kV environ. Le rendement de neutrons fut évalué d'après l'activité de la β-radiation, emmanée par le rhodium irradié à l'intérieur d'un bloc de paraffine aux dimensions $3.5 \times 6.5 \times 12.5$ cm. L'activité du rhodium fut mesurée à l'aide d'une chambre d'ionisation. La puissance de la radiation des neutrons dans la région de la chambre adhérante à la cible de lithium, qui fut d'abord irradiée par un courant de deutons de valeur d'un μA, passé préliminairement à travers un écran d'aluminium 2 microns d'épaisseur,—cette puissance fut trouvée pareille à celle de la source de (Rn+Be) dont l'intensité est 3—5 curies. Cette valeur est obtenue dans des expériences comparatives dans lesquelles la source (Rn+Be) était installée près de la cible de lithium. En augmentant la pression de l'hydrogène pesant dans la chambre nous avons, comme il a déjà été mentionné, observé une augmentation remarquable dans le rendement de neutrons, qui dans le cas optimum atteignait des valeurs équivalentes à 10—20 curies de (Rn+Be). Il est à noter qu'avec un tel régime du travail de la chambre, la radiation des neutrons ne dépend aucunement de la cible de lithium. La cible de rhodium, irradiée aux divers points de la jante de la chambre, donnait la même activité. Les neutrons de cette radiation diffuse sont d'une énergie comparativement petite. Ainsi, cette radiation ne produit pas la réaction $^{27}Al(n, \alpha)^{24}Na$.

Nous avons supposé que la radiation diffuse des neutrons observée est causée par la désintégration (D+D) ayant lieu dans les plans d'électrodes sur lesquels l'hydrogène pesant avait été absorbé. Pour confirmer cette supposition on peut encore indiquer deux particularités caractéristiques du travail de notre cyclotron dans le régime assurant une émission intense d'un flux

de neutrons. On a trouvé que l'intensité de cette radiation ne varie que très peu si les conditions de l'accélération des ions en cas de résonance ne sont pas assez rigoureusement observées. Si pour obtenir un courant d'ions sur la cible (à une longueur d'onde du générateur de haute fréquence donnée) il était nécessaire de maintenir la constance du champ magnétique avec une précision de 0.2%, la précision de 2—3% suffisait pour obtenir une radiation diffuse de neutrons.

De plus, l'intensité du flux diffuse ne dépendait presque pas de la valeur du champ magnétique, c'est-à-dire de cette énergie limite, que le deuton pourrait recevoir dans la chambre, si la résonance de haute fréquence était maintenue avec une précision suffisante.

Toutes les observations prouvent qu'avec notre régime de travail de la chambre les deutons ne font que quelques tours pour atteindre une énergie de plusieurs centaines de kV. Les valeurs de l'activité obtenues indiquent la présence de très grands courants d'ions circulant à l'intérieur des électrodes. En comparant les rendements de neutrons que nous avons obtenus avec la réaction (D+D) aux rendements indiqués dans d'autres ouvrages pour cette réaction nous devons admettre que les courants d'ions atteignent au moins la valeur de plusieurs mA. Il est connu de la littérature récente que les puissances de la radiation de cyclotron admises autrefois sont considérées maintenant comme sous-estimées. Une sonde, située entre les électrodes, a donné la possibilité d'enregistrer les courants d'ions de l'ordre des centaines de μA, tandis qu'à la sortie du système ce n'étaient que des dizaines de μA.

Les résultats de notre travail nous permettent de conclure que des courants encore plus grands, de l'ordre de mA, circulent dans le système des électrodes sans pouvoir être révélés par la sonde. Il en est évident que par un choix convenable des conditions de l'expérience on pourrait obtenir une puissance du rendement des réactions nucléaires beaucoup plus grande que celle qui est connue des observations et des expériences antérieures. On y peut parvenir, par example, en faisant usage de la chambre d'accélération travaillant avec deutérium aux grandes énergies des deutons (de l'ordre de 5—10 MeV), et en couvrant la surface intérieure des électrodes d'une pellicule de glicinium.

Il est bien possible que les hauts rendements de neutrons, obtenus par Lawrence (1) avec sa grande chambre, sont liés à la radiation diffuse des plans intérieurs des électrodes. Cette supposition explique peut-être la différence entre les données de Lawrence et celles d'Amaldi, de Tuve etc. (2), et permet d'éviter les complications théoriques liées à l'admission qu'un des 17 deutons tombant sur une plaque de glicinium produit la désintégration du noyau de cet élément.

Pour conclure notons qu'il paraît possible de construire un cyclotron avec des aimants plus faibles et des chambres aux dimensions beaucoup plus petites, donnant néanmoins un puissant rendement de neutrons. De nos observations concernant les conditions, dans lesquelles l'émission des neurons est créée, il suit que ce n'est que le long de la fente entre les électrodes de la chambre que l'on doit assurer les dimensions plus ou moins grandes de l'aimant.

Il nous reste à remercier les collègues Djelepow et Mostitsky qui nous ont beaucoup aidé dans notre travail.

Institut de Radium.

Maniscrit reçu
le 9. V. 1939.

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PHYSICS

SHAPE OF β-RAYS SPECTRA AND THE MASS OF NEUTRINO

By L. A. SLIV

(Communicated by V. A. Fock, Member of the Academy, 3.V. 1939)

1. Measurements of the shape of β-ray spectra carried out with great care by many authors ([1, 2, 3]) have shown Konopinski-Uhlenbeck's (abbreviated in the following as K-U) formula for the distribution of energy of the emitted electrons ([4]) to differ greatly from the experimental data obtained in the region near the upper limit of the spectrum. For the other part of the spectrum, the K-U formula is in satisfactory agreement with experimental data. The attempt to correct the K-U formula by introducing a mass of neutrino different from zero and to eliminate thereby the divergence near the upper limit, failed to give satisfactory results, since for each β-spectrum a different value for the mass of neutrino was obtained. Furthermore, it was made evident that K-U's theory does not show a correct dependence of the lifetime of heavy β-radioactive elements on disintegration energy ([5]), and leads to too great a relative probability of a K-electron being captured by the nucleus ([6]).

2. The advantage of the interaction law of the K-U form consists in that it gives a necessary asymmetry of the β-ray spectra which is not given by the Fermi interaction law. It is, therefore, necessary to change the K-U law so as to eliminate the divergence between theory and experiment thus arisen.

Konopinski and Uhlenbeck, as well as Fermi, have applied the «Vector» form of the interaction law. Apart from the «vector» form two other forms are possible—the «pseudo-vector» and the «tensor» ([7]). If the mass of neutrino is taken equal to zero, all the three forms give (irrespective of the usual neglects) the same energy distribution of the emitted electrons. Yet, it is not the case when the neutrino mass differs from zero.

The pseudo-vector and tensor containing the wave functions of the electron and the derivatives of the neutrino wave functions can be written as follows:

$$V_k = \psi^* i \rho_2 \frac{\partial \varphi}{\partial x_k} \qquad (k = 1, 2, 3, 0) \qquad (1)$$

$$V_{ik} = \psi^* \rho_3 \gamma_i \frac{\partial \varphi}{\partial x_k} \qquad (i, k = 1, 2, 3, 0) \qquad (1')$$

$\gamma_0 = -\rho_3$, $\gamma_l = \rho_3 \alpha_l$, $(l = 1, 2, 3)$, α_l, ρ_2 and ρ_3 are the Dirac matrices; ψ is the electron wave function, φ that of the neutrino. The square of the matrix element H_β entering the expression for the probability of β-disin-

tegration in case of the «pseudo-vector» or «tensor» form of the interaction law is:

$$|H_\beta|^2 = g^2 \left| \int \left\{ (u_n^* \sigma u_m)(\psi^* i \rho_2 \operatorname{grad} \varphi) - (u_n^* \gamma_5 u_m) \left(\psi^* i \rho_2 \frac{\partial \varphi}{c \partial t} \right) \right\} d\tau \right|^2 \quad (2)$$

or

$$|H_\beta|^2 = g^2 \left| \int \left\{ (u_n^* \rho_3 \sigma u_m)(\psi^* [\alpha \operatorname{grad} \varphi]) + \right. \right.$$
$$\left. \left. + (u_n^* \rho_3 \alpha u_m) \left(\psi^* \alpha \frac{\partial \varphi}{c \partial t} - \psi^* \operatorname{grad} \varphi \right) \right\} d\tau \right|^2, \quad (2')$$

where u_m and u_n are the Eigenfunctions of the nuclei before and after the disintegration, σ is the Pauli spin operator, $\gamma_5 = \gamma_0 \gamma_1 \gamma_2 \gamma_3$, g is a dimensionless constant [as regards the introduction into the Hamiltonian of the derivative of the neutrino wave function with respect to time, see the paper by Fierz (8)]. Neglecting under the integral the terms of the order $\frac{v}{c}$, where v is the velocity of the heavy particles, and taking ψ and φ as constant in the nucleus, we obtain that (2′) differs from (2) by a factor 2 only. In the following we will, consequently, make no distinction between the matrix elements (2′) and (2).

The probability of a β-emission in which the electron receives an energy between E_e and $E_e + dE_e$, is then equal to:

$$P(E_e)\, dE_e = g^2 \left| \int u_n^* \sigma u_m d\tau \right|^2 [1 + k/(E_e E_\nu)]\, F(Z, E_e)\, E_e p_e E_\nu p_\nu^2 dE_e$$

where $E_e E_\nu p_e p_\nu$ are energy and momentum of electron and neutrino respectively; $k = \frac{\mu}{m}$, where μ is the mass of neutrino, m is the mass of the electron. The energies are expressed in units mc^2, the momenta in units mc. $F(Z, E_e)$ represents the influence of the Coulomb field of the nucleus on the energy distribution; for $Z = 0$, $F(Z, E_e) = 1$. In contradistinction to the corresponding K-U formula, we have $E_\nu p_\nu^3$ instead of the factor $E_\nu^3 p_\nu$.

Comparing the distribution given by (3) with the experimental data, we come to a conclusion that the best agreement is obtained when the mass of neutrino is nearly equal to the electron mass. If we take the mass of the neutrino equal to that of the electron, the distribution function in energy $P(E)$ will be

$$P(E) = g^2 \left| \int u_n^* \sigma u_m d\tau \right|^2 (1 + [E(E_0 - E)]^{-1})\, F(Z, E)\, E (E^2 - 1)^{\frac{1}{2}} \cdot$$
$$\cdot (E_0 - E)[(E_0 - E)^2 - 1]^{\frac{3}{2}}, \quad (4)$$

where E is the energy of the emitted electrons (here and in the following the sign e is omitted); E_0 is the energy of disintegration; $E_0 = E_{\max} + 1$. Near the upper limit $P(E)$ decreases as $\sim (E_{\max} - E)^{\frac{3}{2}}$, i. e. in this region of the spectrum the distribution according to (4) is near to the distribution given by the Fermi theory (9). The matrix element $\left| \int u_n^* \sigma u_m d\tau \right|$ gives the selection rules proposed by Gamov and Teller (10).

3. Let us now compare the distribution according to (4) with that obtained from measurements of the β-ray spectra especially produced near the upper limit. Fig. 1 gives the comparison of the experimental spectra RaE (2) and ThC (3) with the distribution (4) near the upper limit. The upper limit

of the theoretical curve for RaE is only by 5 ekV higher than the experimental one, and is equal to 1185 ekV. The upper limit of the theoretical curve for ThC is taken equal to 2220 ekV. It is evident that the theoretical curves are in good agreement with the experimental points, and give correct values for the upper limits of the spectra. In Fig. 2 the curve *I* is the K-U plot for P^{32} ([1]). The theoretical curve calculated after (4) differs but slightly within a certain interval from a straight line. This conforms with a well-known fact that the experimental points of the middle region of the spectra on the K-U plot are placed on a straight line. Curve *II* of Fig. 2 is a Fermi plot for the same spectrum; on this graph the theoretical curve calculated after (4) develops into a straight line near the upper limit. It should be noted that for distribution (4) the Fermi and K-U plots lose their former significance. In Fig. 3 are given the theoretical and experimental spectra for heavy elements RaE ([14]) and ThB ([15]), and for the light elements N^{13} ([13]). For the upper limits of these spectra in (4) are taken the following values: RaE$\rightarrow E_m$=3.32, ThB$\rightarrow E_m$=1.75, $N^{13}\rightarrow E_m$=3.4. A small divergence between the theoretical and experimental spectra of ThB and RaE is observed in the region of small energies where the experimental measurements have as yet not been sufficiently accurate [note that for $F(Z,E)$ in (4) the approximate value from Fermi's paper ([9]) is used. It reduces the accuracy of the theoretical curves in the region of small energies].

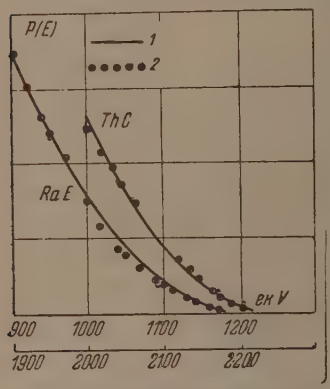

Fig. 1. *1*—theoret. measurments. ... *2*—exper. measurments.

4. The lifetime τ of the β-radioactive elements is determined from the formula

$$\frac{1}{\tau}=\lambda=\int_{1}^{E_m} P(E)\,dE \qquad (5)$$

where $P(E)$ has the form of (4).

In Fig. 4 are given the curves of dependence λ (in sec^{-1}) upon E_m—1 (in electron-volts) for light elements according to the Fermi (curve *I*) and the K-U (curve *III*) theories, and according to the formula (5) (curve *II*).

For great values of E_m—1 the curve *II* will be parallel with curve *III*, while for small ones it runs near to curve *I*. Constructing such curves for heavy elements, we obtain an analogous picture.

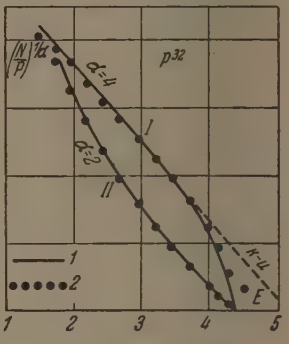

Fig. 2. Symbols as above.

The comparison of the dependence of the lifetime on the maximum energy following from K-U's theory with experimental data shows that for heavy nuclei with a relatively small disintegration energy, as, e. g., ThB and RaD, the K-U theory gives highly exaggerated values for the lifetime τ ([5]) For very light nuclei K-U's theory gives approximately correct values for τ. Fig. 4 shows that for very light nuclei formula (5) gives correct values of τ, too, and may possibly give also correct values of τ for heavy nuclei. The constant g^2 in (4) is $\sim$10 times smaller that the corresponding constant in the K-U theory.

5. The theory of K-electron capture is developed by Yukawa and

Sakata ([11]) and by Möller ([12]). When a matrix element (2) is used and is assumed equal to $\mu = m$, the probability of capture is

$$P_k = g^2 \left| \int u_n^* \sigma u_m d\tau \right|^2 \frac{4\pi (Z\alpha)^3}{\Gamma(3+2S)} \left(\frac{2\rho}{a_0}\right)^{2S} r^{-\frac{2\rho}{a_0}} \cdot$$

$$\cdot (1 - [E_k (E_0 + E_k)]^{-1}) (E_0 + E_k) [(E_0 + E_k)^2 - 1]^{\frac{3}{2}}. \quad (6)$$

Here E_k is the energy of the electron on the K-level expressed in units mc^2, a_0 is the radius of the Bohr orbit measured in units $\frac{h}{mc}$, $\alpha = \frac{1}{137}$, $S = \sqrt{1-(Z\alpha)^2} - 1$, ρ is the radius of the nucleus, E_0 is the energy liberated by the nucleus.

For the comparison of the theory with the experiment the cases are essential where there exists, simultaneously with the probability of capture, a finite probability of a β-ray disintegration with the emission of a positron. In the table are given data for the ratio of the K-capture probability P_k to the positron emission probability λ, for light elements, according to the Fermi and K-U theories and after the formulae (6) and (5).

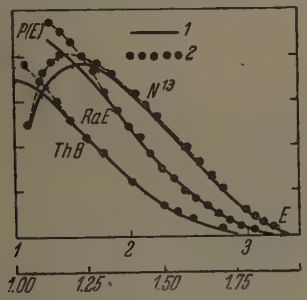

Fig. 3. Symbols as above.

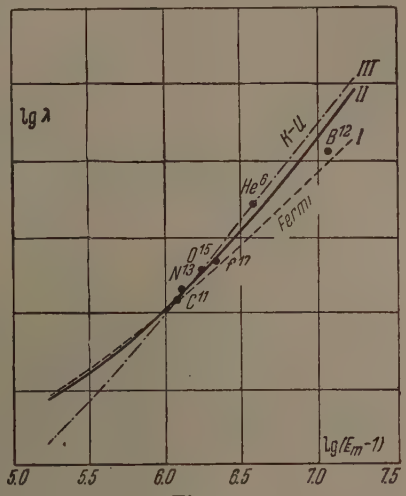

Fig. 4.

$E_0 - 1$		1.118	1.4142	2.29	3.20	4.11	5.02	7.06
$\frac{P_k Z^{-3} 10^5}{\lambda_+}$	Fermi	9000	123	3.20	0.61	0.22	0.10	0.03
	K-U	~1.5 · 10⁶	8730	50.9	5.60	1.55	0.60	0.15
	Formulae (6) and (5)	5000	122	5.12	1.25	0.47	0.24	0.09

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GEOPHYSIK

DER VOLLE STROM DER DRIFTSTRÖMUNG IM MITTLEREN UND SÜDLICHEN TEIL DES KASPISCHEN MEERES

Von W. STOCKMANN

(Vorgelegt von J. M. Schokalski, Mitglied der Akademie. d. 25. IV. 1939)

In Meere gibt es, wie bekannt, drei Arten von unperiodischen Strömungen: Drift-, Gradient- und Konvektionsströmungen. In einem geschlossenen und geschichteten Wasserbecken, wie es das Kaspische Meer ist, finden sich die erwähnten Strömungen gleichzeitig; sie sind übereinander geschichtet und bilden einen komplexen Strom. Theoretisch kann jede der erwähnten drei Arten von Strömungen qualitativ und quantitativ mit Hilfe der bekannten hydrodynamischen Methoden einzeln untersucht werden. Leider können diese Methoden in der Praxis nur mit gewissen Einschränkungen angewandt werden, die ihre Anwendung unter den Verhältnissen des Kaspischen Meeres zu Zeiten ausserordentlich erschweren, schon ganz zu schweigen davon, dass in der Praxis noch grössere Schwierigkeiten bei der analytischen Erforschung des komplexen Stroms bestehen.

Unsere vorliegenden Ausführungen gelten einer der Komponenten des komplexen Stroms im Kaspischen Meer, jener rein driftenden Strömung, die in der gesamten horizontalen Zirkulation der Wassermasse dieses Binnenmeeres keine geringe Rolle spielt. Bei Betrachtung dieser Driftströmung des Kaspischen Meeres als isoliert von den anderen Komponenten des komplexen Stroms, kann das Feld der Driftströmung nach zwei verschiedenen Methoden untersucht werden. Die eine Methode besteht in der Aufzeichnung der Stromlinie der Windströmung, z. B. für die Meeresoberfläche. Zu diesem Zweck muss man nicht nur die absolute Geschwindigkeit der Windströmung kennen, die auf Grund des Windkoeffizienten berechnet werden kann (N. N. Struiski hat diesen für das Kaspische Meer zuverlässig ermittelt), sondern auch den Winkel, den die Oberflächenströmung mit der Windrichtung bildet. Dieser letztere ist leider sehr unbeständig und kann die verschiedensten Werte annehmen, entsprechend dem geänderten Koeffizienten der turbulenten Reibung im Meerwasser, wobei sich auch diese Grösse nur ausserordentlich schwer in zuverlässiger Weise bestimmen lässt. Versuche zur empirischen Bestimmung der Beziehungen zwischen Windrichtung und Richtung der Driftströmung, wie sie in verschiedenen Meeren (darunter auch im Kaspischen Meer) angestellt wurden, haben bisher nicht zu unbestreitbaren zahlenmässigen Ergebnissen geführt. Eben deshalb ist die zweite Methode zur Erforschung der rein driftenden Strömung im Kaspischen Meer, die Methode der Untersuchung des vollen resultierenden Driftstromes zweckmässiger, da dieser in keiner Weise abhängt von den Ände-

rungen der turbulenten Reibung im Meerwasser und vom Mechanismus der Turbulenz.

Laut Ekman bildet die Richtung des vollen Driftstroms im Meere, wenn seine Tiefe die Tiefe der Reibung überschreitet, mit der Windrichtung einen rechten Winkel (der auf der nördlichen Halbkugel rechts vom Winde liegt), während die Grösse S dieses Stroms bestimmt ist durch die Gleichung:

$$S = \frac{\tau_0}{2\omega \sin \varphi} \qquad (1)$$

worin τ_0 gleich dem tangentialen Winddruck an der Meeresoberfläche, φ gleich der geographischen Breite und ω gleich der Winkelgeschwindigkeit der Erdumdrehung ist. Die Grösse des resultierenden Drifrstroms lässt sich errechnen aus den Elementen des Gradient-Windes, wobei man ausgeht von der Verteilung des atmosphärischen Drucks über der Wasserfläche der untersuchten Meeresstelle. Berechnungen dieser Art, wie sie Montgomery für den nördlichen Teil des Atlantischen Ozeans durchgeführt hat, führten zu sehr interessanten Ergebnissen. Indes ist diese Methode, abgesehen von ihrem etwas verwickelten Charakter, weniger Erfolg versprechend als die unmittelbare Berechnung des vollen Driftstromes aus der auf offenem Meer beobachteten Windresultante. Heute verfügen wir für das Kaspische Meer über zahlreiche und langjährige Windbeobachtungen, die in verschiedenen Teilen dieses Binnenmeeres von Schiffen aus angestellt wurden und denen wir die Möglichkeit verdanken, die Elemente der Windresultante für alle 12 Monate des Jahres zu berechnen, wobei diese Elemente auf den Mittelpunkt der Quadrate (mit der Seitenlänge 1°) bezogen werden, in die die Wasserfläche des Kaspischen Meeres zerlegt worden ist. Durchgeführt wurde diese wichtige Arbeit von W. I. Prischlezow.

Ausgehend von den Elementen der Windresultante berechneten wir die vollen Driftströme für den Mittelpunkt eines jeden Quadrats nach Formel (1), in der die Grösse τ_0 bestimmt wurde aus der empirisch bekannten Beziehung:

$$\tau_0 = \rho' \gamma^2 W^2, \qquad (2)$$

wo $\rho' = 1.3 \times 10^{-3}$ die Luftdichte und γ ein dimensionsloser Koeffizient ist, der abhängt von der Rauheit der untergelegenen Wasserfläche und von der Meereshöhe, in welcher die Windgeschwindigkeit (W) gemessen wurde. In unserem Falle fanden wir γ zu 5.45×10^{-7}. Wir wandten Formel (1) auf die jenigen Quadrate des Kaspischen Meeres an, wo wir überzeugt waren, dass in ihnen die mittlere Meerestiefe die obere Reibungstiefe überschritt. Leider haben wir in den vorliegenden kurzen Ausführungen nicht die Möglichkeit, bei der Begründung des von uns gewählten Proportionalitätskoeffizienten in der quadratischen Abhängigkeit zwischen dem tangentialen Winddruck an der Meeresoberfläche und der Windgeschwindigkeit im einzelnen zu verweilen. Raummangel ist es auch, der uns die Möglichkeit nimmt, alle 12 Karten vorzuführen, auf denen die Verteilung der vollen Driftströmung während der einzelnen Monate angegeben ist, wie wir sie auf Grundlage der Windresultante aufgezeichnet haben. Wir beschränken uns hier auf eine allgemeine Charakteristik der von uns gefundenen Ergebnisse.

Die vollen Ströme der Driftströmung im Kaspischen Meer wurden in Kubikdezimetern pro Sekunde ausgedrückt und stellen die Wassermenge dar, die einen Dezimeter breiten, zu dem resultierenden Strom senkrecht stehenden Querschnitt passiert, dessen Tiefe gleich ist der Reibungstiefe (etwa 25 Meter). Auf den von uns aufgezeichneten Karten wurden die vollen Ströme als Vektoren dargestellt, die an den Zentren der entsprechenden Quadrate angelegt wurden. Auf Grund der Elemente des vollen Stroms der

Windströmung wurden dann die Grössen der Konvergenz oder Divergenz des Stroms für jedes Quadrat pro Dezimeter und Tag errechnet. Zur Illustrierung des Gesagten wurde in Abb. 1 ein Schema der Stromlinie der Windresultante über dem Kaspischen Meer entworfen und in Abb. 2 eine Karte der Verteilung der vollen Driftströme im mittleren und südlichen Kaspischen Meer für die gleiche Periode abgebildet. Der Abb. 1 sind ausser der

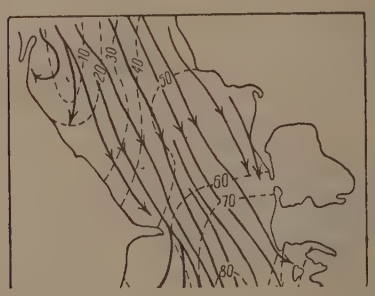

Abb. 1.

Abb. 2.

Stromlinie der Windresultante die Isolinien der Beständigkeit der Windrichtung in Prozenten beigefügt. Die Divergenz-Gebiete in Abb. 2 sind durch eine punktierte Linie kenntlich gemacht; die Zahlenwerte der Divergenz (mit Minuszeichen) und der Konvergenz (mit Pluszeichen) sind zu Dezimetern pro Tag abgerundet und in Kreisen im Zentrum jedes Quadrates angebracht.

In den Fällen, wo sich der volle Driftstrom als so gross erwies, dass er nicht durch einen kontinuierlichen Vektor in dem angenommenen Masstab dargestellt werden konnte, wurde der Strom durch einen kleineren Vektor mit Angabe der wirklichen Länge des letzteren in Millimetern (1 Millimeter=1 Kubikdezimeter pro Sekunde) dargestellt.

Abb. 3.

Auf die Grösse der Vertikal-Komponenten der Strömung, die durch Divergenz oder Konvergenz im Felde des resultierenden Driftstroms bedingt sind, kann annähernd nach der Grösse der Divergenz oder Konvergenz gegchlossen werden, die in Abb. 2 dargestellt sind.

Die Betrachtung der von uns zusammengestellten Karten hat uns zu der Schlussfolgerung geführt, dass die Gebiete der Divergenz sich vor allem (im Laufe eines Jahres) am Ostufer des mittleren und südlichen Kaspischen Meeres befinden, während die Gebiete der Konvergenz hauptsächlich der westlichen und südlichen Küste des Meeres anliegen. In den Sommermonaten (insbesondere im Juli) macht sich die Divergenz am Ostufer des mittleren Kaspischen Meeres am meisten bemerkbar. Die Divergenzkompensation oder die Geschwindigkeit der dort aufsteigenden Strömungen—unter der Bedingung, dass die tieferen Wasserschichten unbeweglich sind—erreicht im Juli in einigen Quadraten die maximale Grösse. Es ist offensichtlich, dass die Folge der Divergenz am Ostufer des mittleren Kaspischen Meeres eine

verhältnismässige Abkühlung der obersten Wasserschicht im Sommer sein muss, was ausgezeichnet durch die Karten der durchschnittlichen Monatsisothermen an der Oberfläche des Kaspischen Meeres bestätigt wird. In Abb. 3 sind die Juliisothermen für den mittleren Teil des Meeres dargestellt. In der Tat macht sich, wie zu erwarten war, der Unterschied zwischen der Wassertemperatur am Ost-und Westufer des mittleren Kaspischen Meeres besonders stark im Juli bemerkbar, eben in demjenigen Sommermonat, wo die Divergenz im Driftstrom ihr Maximum erreicht und die Windrichtung am beständigsten ist. (Abb. 1 und 2). Ferner ist eine sehr interessante Tatsache noch zu erwähnen. Das beständige Divergenz-Gebiet am Ostufer des mittleren Kaspischen Meeres ist identisch mit dem den Hydrobiologen wohlbekannten Küstengebiet der lebhafteren Bodenproduktion (Benthos). Dieses Zusammenfallen ist schwerlich ein Zufall. Beständige aufsteigende Strömungen können einen Faktor bilden, der die Entwicklung der Lebewesen auf dem Meeresboden in dem erwähnten Gebiet begünstigt, indem sie die obere Wasserschicht durch die aus tieferen Schichten stammenden lebensbegünstigenden Elemente bereichert. Nach den zuverlässigsten Berechnungen beträgt die Gesamtwassermenge, die im Juli am Ostufer des mittleren Kaspischen Meeres aus den unterhalb der Reibungstiefe liegenden Schichten aufsteigt, etwa 231 Kubikkilometer, eine Menge, die nur wenig zurückbleibt hinter dem mittleren Jahreszufluss an Wolgawasser (261 Kubikkilometer). Die angeführten Zahlen bedeuten wahrscheinlich das Maximum den Gesamtwassermenge, die in dem erwähnten Zeitraum am Ostufer des Kaspischen Meeres aus tieferen Schichten nach oben steigt.

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GEOPHYSICS

EFFECT OF ELECTRIZATION OF EARTH LAYERS BY ELASTIC WAVES PASSING THROUGH THEM

By A. G. IVANOV

(Communicated by O. J. Schmidt, Member of the Academy, 9. V. 1939)

Experiments on the application ([1,2]) of the so-called seismo-electric effect for recording oscillations are well-known in the practice of seismic prospecting for mineral wealth; in these experiments oscillograms are taken of variations in amperage due to the action of trembling, the current being introduced into the earth by means of two or several electrodes. American geophysicists ([1,2]) maintain that this effect is manifest within the rocks (particularly in the immediate vicinity of the electrodes), while other investigators ([3]) infer from laboratory experiments that the seismo-electric effect is called forth by electro-motor-chemical processes taking place on the surface of electrodes immersed into an electrolytic solution (primary stratum waters); it should be noted that the latter phenomena were previously studied by Kistiakowsky ([4]) and others.

In the summer of 1938 the author of the present article succeeded in ascertaining the existence of a new, hitherto unknown effect of electrization of rocks upon the passage of elastic oscillations through them, which effect with respect to its manifestations and industrial possibilities was found to be radically different from the above-mentioned seismo-electric effect with current.

If two electrodes (so-called non-polarizing electrodes*) be set up in earth at a certain distance apart and if an explosion be produced at some distance therefrom, then, upon the approach of the elastic wave to the zone surrounding the electrodes, there will appear in the rocks between the electrodes a variable potential difference; and by oscillographing the variations of this potential difference it will be possible to obtain a record resembling an ordinary seismogram.

The investigations conducted by the author have actually proved the existence of a distinct effect of rock electrization, this effect being different from the ordinary seismo-electrical effect with current. Let us term conventionally the formerly known seismo-electrical effect with current as the [*J*-effect] and the electrization effect discovered by us as the «*E*-effect».

I. Experiment 1 served to prove that the *E*-effect under consideration is manifest within the volume of rocks and not at the boundary of the interface: electrode—earth.

* Use was made of electrodes consisting of copper tubes placed in porous vessels made of special clay; the electrodes were immersed in a saturated solution of blue copperas.

Next to that electrode which was nearest to the spot of the explosion an electric seismograph was set up; whereupon the explosion was made. A scheme of the reiterated experiments is given in Fig. 1.

It was found that electrical oscillations in the circuit of the electrodes begun, as a rule, earlier than in the circuit of the seismograph (in our case 1 to 2 hundredth fractions of a second earlier).

In Fig. 2 is to be seen one of the seismograms, illustrating the aforesaid. The curve corresponding to the circuit of the electrodes is denoted by the

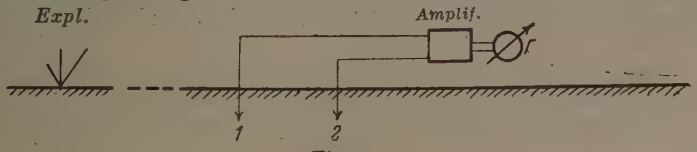

Fig. 1.

letters *MN*; time marks (transverse lines) are plotted every 0.01 fraction of a second. If we assume the velocity of the longitudinal elastic waves in the upper stratum to be: $a_1 = 1000$ m/sec., and the gain in velocity over the first contribution to be: $\Delta t = 0.015$ sec., then in this particular case «the catching radius» of the set will be $\Delta ta = 0.015 \cdot 10^3 = 15$ m. Every time control observations were made in order to become convinced that the trembling and secondary electric circuits did not influence the receiving apparatus directly.

Experiment 1 showed conclusively that the effect of electrization takes place within the volume of rocks, wherever the elastic wave is propagated. At the same time the electrodes served in first place to catch the electromag-

Fig. 2.

netic field running in front of the elastic wave and decreasing with distance.

The appearance of the records and the amplitude of electrization are evidently mainly determined by the elements of the elastic wave and the properties of the rocks in the given locality.

II. Experiment 2. It might have been supposed that effect *E* under consideration is none other but the known seismo-electric effect *J*, but only displayed in the field of natural earth currents. In order to throw full light upon this question, the author produced the explosion first on one side of the pair of receiving electrodes (along the line connecting them) and then on the other side (along the same line). All the rest of the set-up remained unchanged during the whole of the experiment. In both these cases the oscillograms proved to resemble one another in substance except that the «phases» of the natural oscillations varied 180° upon transfer of the

explosion spot to the other side of the electrodes. While with an explosion «to the right hand» from the set-up, the first great oscillation ran upon the recording tape in an upward and then in a downward direction, with an explosion «on the left» the course of this curve was found to be reversed.

It is to be remarked that in observation of the familiar seismo-electric effect J (with current) the «phase» of oscillations upon such a transfer of the spot of explosion from one side to another remains, as should be expected, unchanged.

Thus, experiment 2 showed that the new effect of electrization E differs from familiar effect J.

Apart from the above-described two fundamental and many times reiterated experiments, investigations were also conducted for the purpose of elucidating certain peculiarities of the E-effect, as well as of the different phases of its recording. It should be noted that up to the present time the desired sensitivity of the set-up has not yet been attained (for lowering the magnitude of the applied explosions); and neither has E-effect been investigated in any other districts outside of the Bashkir ASSR where the original experiments were conducted.

The author believes that the electrization of the strata of the earth concomitantly with trembling may be due to availability in rocks (seen as a compound disperse system) of such solid and liquid phases with a large inner surface which cause a complex of e l e c t r o - k i n e t i c phenomena in this system.

In rocks, at the boundary line of the interface between solid particles and primary water, there exists, as is known, a so-called double diffusion layer. This is formed because the surface of particles adsorbs ions of a definite sign out of the solution. One portion of the interlayer, namely that which faces the liquidis mobile. Now, the author maintains that with elastic compression or distention, the thicknesses of the double diffuse layers between the particles undergo a change at the respective spots, this being brought about by the falling together or the mutual dissociation of these particles. In course of this, it is to be supposed that in the respective sections the force of the electrical action exerted by the solid particles upon the ions in the double layer varies too, and part of these particles, notably those of one definite sign, temporarily becomes either liberated or else fixed; thanks to which the potential of the electrolyte pervading the rocks likewise becomes temporarily changed; in result a p. d. is set up between quiescent spots and places where the wave is propagating at the given moment. Upon the passage of the elastic wave the particles of the rocks return to their original position and the thicknesses of the double diffuse layers become as they had been previously.

Further, it may, for instance, be admitted that in course of the propagation of the elastic wave a relative displacement of the electrolyte solution and the solid particles will take place (analogous to the squeezing out of a liquid by an elastic porous body during compression). In course of this there will arise in the rocks so-called filtration potentials (flow potentials) whose instantaneous magnitudes are, according to Helmholz, to be determined by the velocity of relative displacements.

The possibility is not excluded that other mechanisms arising in solid and liquid bodies concomitantly with pressure and trembling may also contribute their share in bringing about the E-effect.

The E-effect in question may be serviceable in studies of the physical properties of rocks.

Its application for the prospecting of mineral wealth (p r o v i d i n g f o r a f a v o u r a b l e c o u r s e o f f u r t h e r i n v e s t i g a t i o n s

which would result in an increase of the sensitivity of the receiving scheme) may offer definite advantages as compared with existing methods. The use of the *E*-effect may afford the possibility of resorting to seismic prospecting by reflected waves with records of oscillations taken at a definite «point» by means of a single seismograph; and also of recording the integral effect within the thickness of rocks, which would eliminate troubles caused by the mellow surface of the earth, this being a factor which considerably impedes prospecting in a number of districts, for instance in the Ishimbaev district of the Bashkir ASSR.

By means of the application of several electrodes and through their special location it would in all probability be possible to accomplish directional reception of seismic oscillations. Observations can be set up with a vertical position of the electrodes, this being done with a view to carrying into practice the electrization method of taking boring samples, so that the best possible information may be obtained regarding the physical properties of the rocks surrounding the bore-hole.

Finally we may surmise that the electrical phenomena occurring in the earth's crust during earthquakes are associated with the electrization effect.

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CHEMISTRY

HYDROGENATION OF WASTE SULPHITE LIQUORS

By P. A. BOBROV and L. I. KOLOTOVA

(Communicated by A. E. Favorsky, Member of the Academy, 11. V. 1939)

When cellulose is manufactured from wood by sulphite process, there is wasted about 5% of wood by way of soluble-in-water compounds formed by certain components of wood as, for instance, lignin and hemicelluloses with calcium sulphite. These aqueous solutions, called waste sulphite liquors, are obtained in several million tons and drained off into rivers and other reservoirs of public use, polluting the water in them and, without any exaggeration, becoming a social scourge in this way. The problem of utilization of sulphite liquors is, therefore, highly actual.

The work undertaken by the authors is, as far as it is known to them, a first attempt to apply hydrogenation to reforming of waste sulphite liquors.

Oxygen (about 30%) and sulphur (up to 7%) constitute part of the organic material of waste sulphite liquors, beside carbon and hydrogen. The former two are completely eliminated during exhaustive hydrogenation, whereby great ammounts of hydrogen are used up.

When waste sulphite liquors are hydrogenated, certain peculiar relations are observed which fail to appear in a common, destructive hydrogenation. A consumption of hydrogen is for the first time observed in a temperature range from 140 to 170°, the reducing substances of waste sulphite liquor being reduced herewith. During further heating to approximately 280—300° the consumption of hydrogen is inappreciable; on the contrary, there is observed a larger increase in pressure than such as might be put down to the rise of temperature only. But heating over 300° makes the consumption of hydrogen appear again.

High yields of heavy oils in an almost complete absence of light fractions were obtained when heating was continued for 5 hours at temperatures not exceeding 340—350°. A sharp increase in the field of light fraction, boiling below 150°, was attained by bringing the temperature above 400°.

It would seem that the lenght of exposure to temperatures ranging between 300 and 320° is a factor in the total yield of hydrogenation products (oils).

The method of obtaining the initial substance for hydrogenation experiments was to subject an aqueous solution of waste sulphite liquors acidified with sulphuric acid to steam-treatment at 100° and then to evaporate it to dryness. With such a treatment a half of the fixed sulphur is removed from the solution in the form of sulphur dioxide, and the latter can easily be turned

to account in cellulose manufacture. This preliminary treatment leads to a considerable saving of hydrogen. The yield of the initial material amounted up to 70.5% of the weigth of the compact dry residue obtained through direct evaporation of waste sulphite liquors, or to 7.05% from the weight of the cooking liquor. The initial material thus obtained was blended with heavy hydrogenation oil in a proportion of 2 : 1. On account of the conditions available it was necessary to start with a low initial pressure of hydrogen—about 34 to 40 atm.—and to take a small portion of material, viz. 80 g, the capacity of autoclave being 2.2 l. The time of heating at 350—450° was 6 hours on an average.

There were obtained the following yields of the hydrogenation products: the total yield of oils was 55% by weight on the average with respect to the initial portion tested. The yield of fractions boiling below 150° amounted to 40% of the total amount of oils, or 24% with respect to the weight of the initial portion when the hydrogenation temperature was kept below 480°. When it was brought below 350°, the yield of light fraction did not exceed 8% of the total amount of oils and 4.8% of the initial portion. The average amount of solid carbonaceous residue left after hyrdogenation was 18% of the initial portion, the ash content in the residue being 50%.

The gas mixture left after hydrogenation contained a large amount of CO_2.

In the numerous works on hydrogenation of wood, lignin, cellulose, wood dry distillation tar etc., the hydrogenation was carried out according to the general methods of high temperature destructive hydrogenation, thus leading always to extremely high expenditure of hydrogen. The natural conclusions the authors arrived at was that the hydrogenation of such products is not renumerative. This conslusion finds support in the ultimate composition of these meterials which contain a high percentage of oxygen.

Various substances, besides the waste sulphite liquors, as, for instance, wood, lignin, cellulose, wood dry distillation tars and other products, are known to break down at temperatures about 270—280° and for some of them an exothermity of this reaction was demonstrated.

Consideration of the phenomena involved in a thermal decomposition of the above-mentioned substances reveal some very important points in the hydrogenation of these products.

A thermal decomposition of materials referred to above is accompanied with an evolution of considerable amounts of water, carbon dioxide, carbon monoxide, etc.

The mechanism of formation of these latter compounds is not clearly ascertained as yet. A most probable conjecture supported with some experimental data is the one which assumes that the liberation of water and carbon dioxide is a result of decomposition of hydroxy-acids formed as intermediate products of thermal decomposition of carbohydrate substances. Be that as it may, we have in all of those cases a liberation on fixed oxygen from organic material in the form of water and carbon dioxide. Hence a conclusion to be drawn is that the calculation of hydrogen consumption on the basis of ultimate composition of the hydrogenated compound is inapplicable when the process of hydrogenation is carried out correctly.

Hydrogenation of such materials must be preceded by their thermal decomposition. These considerations gain added convincing strength when applyied to concrete examples. Thus, in the case of wood dry distillation tars containing above 50% of non-volatile hydroxy-acids, the main part of hydrogen consumption is, judging from the studies available in the literature, due to the reduction of hydroxyl and carboxyl groups of these acids. However, the liberation of water and carbon dioxide at the expense of hydr-

oxyl and carboxyl groups is brought about on simple heating to relatively low temperatures attaining a maximum at about 270°.

Thus, a comparatively low preliminary heating is sufficient to remove the main part of the fixed oxygen from the material to be hydrogenated and in this way to lower the consumption of hydrogen considerably. Approximately the same may be said about the hydrogenation of wood and of other products referred to earlier.

The aforesaid relates to a case of destructive hydrogenation of the above-mentioned materials. In many cases the hydrogenation can be carried out with other aims. Thus, dry distillation tars may be hydrogenated with the purpose to reduce hydroxy-acids into fatty acids. In this and other similar cases the hydrogenation should be carried out at temperatures below those of thermal decomposition of the hydrogenated material. Generally speaking, a proper notion of the process of thermal decomposition of hydrogenated material may afford valuable indications.

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CHEMISTRY

STUDY OF BREAK-DOWN PRODUCTS OF LIGNIN

By P. A. BOBROV and L. I. KOLOTOVA

(Communicated by A. E. Favorsky, Member of the Academy, 11. V. 1939)

In previous communications (1) the authors have proposed a method for treating wood and lignin which is essentially as follows: wood, reduced to fragments and prepared in the usual way, is exposed to preliminary treatment under pressure in an aqueous medium at 180° for two hours. An almost complete removal of hemicelluloses which pass into aqueous solution is achieved under these conditions. The residue left after treatment in an autoclave consists mainly of lignin and cellulose. Upon oxidation, the entire lignin passes into solution in the form of water soluble acids, while the cellulose remains in the insoluble residue. Some of these acid were identified as their phenylhydrazides. In that way the presence of gluconic, xylonic and mucic acids was established.

The subject of the present paper is a further investigation of the products obtained by oxidation of lignin. Ba salts of hydroxy-acids were evaporated from their aqueous solution to yield a Ba salt which on recristallization gave a product of constant composition. The content of Ba in this product was found to be 53.43%. The amount of Ba in tartronic acid salt is 53.8%. The conclusion to be drawn from the identification of hydroxy-acids obtained by oxidation of lignin is that the latter partakes of the nature of carbohydrates.

To verify this important conclusion, we proceeded to identifying the hydroxy-acids by the method of their reduction with hydrogen iodide. The reaction was run with 4 g of Ba salts, 30 ml of hydroidic acid, sp. g 1.19, and 1.4 g of red phosphorus in sealed tubes at 170—180° for 4 hours. The reaction mixture was then diluted with three times its volume of water and repeatedly extracted with ether. Part of the products remained unextracted. The residue left after distilling off ether was treated with 25% sulphuric acid and zinc dust in the cold. On filtration the filtrate was treated with lead carbonate and by the end with silver oxide. The acidified solution was extracted with ether, the latter being distilled off on drying. The residue had a characteristic odour of fatty acids. In this way 42.5 g of Ba salts were treated. The acids were distilled into four fractions: 1) below 125°; 2) 125—165°; 3) 165—185° and 4) 185° C. The acids divided into fractions were converted into *p*-bromphenacylic esters by treating their Na salts with *p*-bromphenacyl bromide (2). The esters obtained were purified by recristallization from aqueous alcohol.

The following results were obtained from the determination of the ultimate composition and melting points.

The derivative obtained from the fraction 1 melted at 86—86.5° C. Accord-

ing to literary data, the melting point of acetic acid ester is 86°. [Cu salts were prepared from fraction 3. The percentage of Cu was 26.45 and 26.47%. Theoretically the percentage of Cu in butyrate is 26.76].

Subst.—0.0951 g; CO_2—0.1623; H_2O—0.0279.

Subst.—0.1080 g; CO_2O—0.1838; H_2O—0.0324.

Found: C—46.58 and 46.39%; H—3.26 and 3.33%. Calc. C—46.69%; H—3.50%.

Subst.—0.1800; AgBr—0.1304. Found: Br—30.83 and 30.62%.

Subst. —0.1920; AgBr—0.1382. Calc.: Br—31.12%.

The melting point of the ester prepared from fraction 2 was 63 and 64°C. In the literature the ester of butyric acid is reported to have a melting point of 63°C.

Subst.—0.1013 g; CO_2—0.1888; H_2O—0.0459.

Subst.—0.0958 g; CO_2—0.1794; H_2O—0.0414.

Found: C—50.84 and 51.04%; H—5.03 and 4.80%. Calc.: C—50.53%; H—4.56%.

Subst.—0.1530 g; AgBr—0.0994. Found: Br—27.66 and 27.38%.

Subst.—0.1545 g; AgBr—0.0994. Calc.: Br—28.07%.

The melting point of the ester prepared from fraction 4, was 74° C. It is 71.5° C for the ester of capronic acid according to literary data.

Subst.—0.1034 g; CO_2—0.2027; H_2O—0.0505.

Subst.—0.0933 g; CO_2—0.1827; H_3O—0.0458.

Found: C—53.48 and 53.38%; H—5.42 and 5.47%. Calc.: C—53.67%; H—5.43%.

Subst.—0.2000 g; AgBr—0.1215. Found: Br—25.85 and 25.56%.

Subst.—0.2300 g; AgBr—0.1382. Calc.: Br—25.56%.

Glycolic acid, isolated as its Ba salt, was found among the non-volatile products of reduction. The Ba salt was converted into Ag salt and the latter was analyzed.

Found: Ag—58.82; 58.95%. Calc. for Ag salt of glycolic acid: 59.02%.

Lignin when oxidized by hydrogen peroxide in a neutral medium passed entirely into solution in the form of salts of the acids produced; some of them was identified. In an alkaline medium, however, some insoluble-in-water products of oxidation were observed to form alongside of the acids.

Lignin was oxidized with 20% hydrogen peroxide at room temperature, the reaction medium being maintained slightly alkaline through the addition of a 2 *N* solution of sodium hydroxide. An aqueous solution of Na derivative of the oxidation products was acidified with hydrochloric acid, whereby a voluminous flaky white precipitate was observed to form. The yield of the latter (rough product) ranged between 54 and 60.7% of the weight of oxidized lignin. The water soluble acids were perfectly similar in their solubility and other properties to those obtained on the oxidation of lignin in neutral medium.

The substance isolated on acidification of the alkaline solution looked white at first, but was gradually acquiring a yellow colour and growing darker and darker with the time, finished with turning black. The change in solubility went parallel with the alteration of colour. At the time when the substance had been obtained it dissolved rather easily in water when heated and did so in equeous solutions of alkalies in the cold; it was readily soluble in methyl and ethyl alcohols, acetic acid, acetone, pyridine, glycerol and other similar solvents. On standing the substance was losing its solubility with respect to above-mentioned solvents and came to a state when it could be dissolved only by means of a treatment with aqueous alkalies. Isolated from an alkaline solution through acidifying the latter, the substance regained its initial solubility.

Attempts to obtain the isolated substance in a crystalline form were not successful and it was purified through conversion into Na derivatives soluble in water and subsequent precipitation by acidifying. After several repeated precipitations the substance had a constant ultimate composition. The material obtained melted with decomposition and did not distill in vacuo.

Purified as described above, the substance gave a slight reaction for furane when heated in glycerol up to the boiling point of the latter; when heating of the dry substance was conducted until signs of a dry distillation became apparent, the reaction for furane was quite distinct.

A treatment of the substance with acetyl chloride in the presence of tin tetrachloride failed to bring about the expected results.

No absorption of hydrogen was observed when Na derivatives were hydrogenated in the presence of Pd. Attempts to sulphonate the substance were a failure. The substance was titrated by 0.1 *N* solution of alkali in the presence of phenolphthalein under usual conditions. Na and Ba salts of the intermediate product of lignin oxidation as well as its bromine derivative have been obtained.

The bromine derivative was prepared through bromination of the substance in glacial acetic acid solution or of its Na derivatives in an aqueous solution. The product of bromination obtained was soluble in methyl and ethyl alcohols and in ethyl acetate. It was purified by precipitation from an alcohol solution. An ultimate analysis of the intermediate product of its Na salt and bromine derivative were carried out.

Intermediate product

Subst.—0.1193 g; CO_2—0.2507; H_2O—0.0550.
Subst.—0.1035 g; CO_2—0.2180; H_2O—0.0485.
Found: C—57.33, 57.44%; H.—5.11, 5.21%. Calc. for C_2H_2O, C—57.14%; H—4.8%.

Determination of metoxyls by Zeisel

Subst.—0.1742 g; AgI—0.1282; OCH_3—9.71%.
Subst.—0.1534 g; AgI—0.1129; OCH_3—9.71%.

Na salt of the intermediate product

Subst.—0.0816 g; CO_2—0.1477; H_2O—0.0261.
Subst.—0.08925 g; CO_2—0.1618; H_2O—0.0299.
Found: C—49.44, 50.52%; H—3.69, 3.72%.
Subst.—0.0970 g; Na_2SO_4—0.038. Found: Na—12.69%.
Calc. for $C_8H_7O_4Na$: C—50.53%; H—3.71%; Na—12.10%.

Bromine derivative obtained by bromination in aqueous solution

Subst.—0.1035 g; CO_2—0.1452; H_2O—0.0234.
Subst.—0.1085 g; CO_2—0.1514; H_2O—0.0279.
Found: C—38.26, 38.06%; H—2.51, 2.86%.
Subst.—0.0922 g; AgBr—0.0710.
Subst.—0.0805 g; AgBr—0.0617.
Found: Br—32.75, 32.61%; Calc. for $C_8H_7O_4Br$: C—38.86%; H—2.86%; Br—32.39%.

The empiric formula which is to be ascribed to the substance in question from the data of analysis referred to above is $C_8H_8O_4$.

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CHEMIE

DIE SOLVATATION VON EIWEISSTOFFEN IN WÄSSERIGEN LÖSUNGEN SCHWACHER SÄUREN UND BASEN

Von W. A. WILENSKI und A. J. KOROLEW

(Vorgelegt von N. D. Zelinsky, Mitglied der Akademie, d. 10. V. 1939)

Die überwiegende Anzahl der Arbeiten, die der Erforschung der Eiweiss-Solvatation gewidmet sind, bezieht sich auf die besondere Erscheinung der Solvatation, wenn Wasser als Lösungsmittel dient (Hydratation) (1). Das interessante und wichtige Problem der Wechselwirkung von Eiweissstoffen mit anderen Lösungsmitteln ist heute noch sehr dürftig erforscht.

Wilenski und Pawlowa (2) untersuchten die Löslichkeit von Phenol, Anilin und anderer Stoffe in Kaseinsolen und fanden, dass die betreffenden Stoffe neben Wasser an der Solvatation von Kasein teilzunehmen fähig sind. Dieser Umstand gab Anlass zu der Vermutung, dass die Wechselwirkung von Eiweisstoffen in wässerigen Lösungen schwacher organischer Säuren und Basen mit letzten sich nicht allein auf Salzbildung beschränkt, wie dies bis zuletzt angenommen wurde, sondern die Bindung der Moleküle des schwachen Elektrolyts zur Folge hat.

Was Essigsäure, Pyridin und andere Stoffe betrifft, so sind Tatsachen bekannt, die die erwähnte Vermutung indirekt bestätigen. So beobachtete z. B. Chiari (3) eine grössere Quellung von Gelatine in Lösungen von Essigsäure, Pyridin und anderen organischen Stoffen im Vergleich zu Lösungen von HCl und NaOH, wenn der Vergleich bei gleicher Aktivität von H^+-Ionen vorgenommen wurde. Während Loeb (4) diese Tatsache in Bezug auf Essigsäure erklärt, nimmt er an, dass die Ursache dieser Erscheinung in der Verringerung der Aggregatkräfte zwischen den Gel-Teilchen unter dem Einfluss der in grosser Menge vorhandenen undissoziierten organischen Säure liegt. Küntzel (5) sprach den Gedanken aus, dass diese Verringerung der Aggregatkräfte durch die Bindung der Moleküle-Säure vom Eiweiss bedingt sein kann.

Das Ziel der vorliegenden Arbeit war die Untersuchung der Wechselwirkung von Kasein mit den binären Lösungsmitteln Wasser-organische Säure. Hierbei beabsichtigten wir, den experimentellen Nachweis zweier Bindungsarten organischer Säuren durch Kasein zu erbringen: einer gewöhnlichen chemischen Bindung der Säure (Bildung von Eiweissalz) und einer molekularen Sorption der organischen Säure, die anscheinend einen Prozess der Proteinsolvatation darstellt.

Als Untersuchungsmethode war die Gleichgewichtsdialyse gewählt (6). Die Zelle wurde mit dem Kaseinsol von innen geladen, aussen befand sich die Lösung der organischen Säure, die zum Kaseinlösen benutzt wurde. An-

fänglich war die analytische Konzentration der Säure C_0 gleich. Während des Prozesses der Einstellung des Dialysengleichgewichts ging ein Teil der Säure aus der äusseren Flüssigkeit in das Eiweissol infolge deren Aufnahme durch das Kasein über. Hierbei ging in der äusseren Flüssigkeit eine Verringerung der analytischen Säurekonzentration C_0 vor sich, welche von uns mit Hilfe eines Interferometers bestimmt wurde. Dieser Effekt könnte erstens durch die Wechselwirkung der H^+-Ionen der Säure mit den basischen Gruppen des Proteins (Bildung von Eiweissalz) bedingt sein; diesen Teil ΔC bezeichnen wir ΔC_S. Zweitens könnte die Abnahme der Säurekonzentration in der äusseren Flüssigkeit auch als Resultat der molekularen Säurenbindung betrachtet werden. Nehmen wir an, dass diese Bindungsart eine Abnahme der Säurekonzentration ΔC_m hervorgerufen hat, dann ist

$$\Delta C = \Delta C_S + \Delta C_m. \tag{1}$$

Da das Ziel dieser Arbeit die Feststellung und die quantitative Bestimmung der molekularen Bindung organischer Säuren durch Kasein war, so wollen wir für diese Bindungsart den Ausdruck ableiten.

Aus der Gleichung (1) folgt

$$\Delta C_m = \Delta C - \Delta C_S. \tag{2}$$

Für die ΔC_m-Bestimmung ist notwendig die Abnahme der Säurekonzentration in der äusseren Flüssigkeit zu kennen, welche durch die salzartige Säurebindung durch Eiweiss ΔC_S hervorgerufen wurde. Dieser Wert kann auf folgende Weise bestimmt werden. Die Messung der H-Ionenaktivitäten zu beiden Seiten der Membran nach der Dialyse ermöglicht, die Konzentrationswerte der Säurenanione sowohl in Sol C_A^I als auch in der äusseren Flüssigkeit C_A^{II} zu berechnen. Gemäss dem Gesetz des Membrangleichgewichts ist

$$a_A^I = \frac{a_H^{II}\, a_A^{II}}{a_H^I}, \tag{3}$$

wo a_H^I und a_A^I die Aktivitäten der H^+-Ionen und der Anionen der Säure innerhalb und a_H^{II} und a_A^{II} — ausserhalb der Membran darstellen.

Nimmt man an [7], dass die Aktivitätskoeffizienten beider Ionen in der äusseren Flüssigkeit gleich sind, so ist

$$a_A^I = \frac{(a_H^{II})^2}{a_H^I}. \tag{4}$$

Während wir zusammen mit anderen Autoren den Aktivitätskoeffizienten der H^+-Ionen im Eiweissol gleich dem Aktivitätskoeffizienten in der äusseren Flüssigkeit annehmen, können wir in der letzten Gleichung von den Aktivitäten zu den Konzentrationen übergehen:

$$C_A^I = \frac{(a_H^{II})^2}{a_H^I f_H}; \tag{5}$$

entsprechend ist

$$C_A^{II} = \frac{a_H^{II}}{f_H}. \tag{6}$$

Indem man die C_A^I und C_A^{II}, das Volumen dar Säurelösung V_1, welches für die Eiweissauflösung benutzt wurde, und V_2 — das Volumen der Säurelösung, das sich ausserhalb der Membran befand, kennt, kann man den Ausdruck für denjenigen Teil der Säurenkonzentrationsänderung in

der äusseren Flüssigkeit finden, der durch die salzartige Bindung und die Donnansche Verteilung bedingt ist,

$$\Delta C_S = \frac{C_A^I V_1 - C_A^{II} V_2}{V_1 + V_2}. \qquad (7)$$

Setzt man ΔC_S aus Gleichung (7) in die Gleichung (2) ein, so erhalten wir den Wert ΔC_m:

$$\Delta C_m = \Delta C - \frac{C_A^I V_1 - C_A^{II} V_2}{V_1 + V_2}. \qquad (8)$$

Die molekulare Sorption der organischen Säure durch das Eiweiss M in mol/g ist also:

$$M = \frac{\Delta C_m (V_1 + V_2)}{g V_2}, \qquad (9)$$

wo g die Eiweisskonzentration in g/l ist.

Die Berechnungen der molekularen Säurensorption durch Kasein wurden nach Gleichung (9) vorgenommen.

Als Untersuchungsobjekt wurde lufttrockenes, nach der Methode von Hammarsten hergestelltes Kasein benutzt. Die Versuche der Gleichgewichtsdialyse wurden mit Essig-, Milch- und Zitronensäure angestellt. Vor dem Versuch wurden die aus Kollodium hergestellten Dialysezellen in Gleichgewicht mit der Säurenlösung bestimmter Konzentration gebracht, zu welchem Zwecke sie in der Säurenlösung drei Tage lang in einem geschlossenen Gefäss gelassen wurden. Nachdem wurde ein Teil dieser Säurenlösung für die Auflösung des Kaseins darin, ein anderer Teil als äussere Flüssigkeit benutzt. Das Volumen des Sols und der äusseren Flüssigkeit war in den Dialyseversuchen gleich (8).

Zu beachten ist, dass die gleichzeitig vor sich gehende Eiweisshydratation den auf dem Interferometer gemessenen Effekt der Säurenkonzentrationsänderung ΔC senkt. Infolgedessen sind die berechneten Sorptionswerte M gegenüber den wahren etwas kleiner.

In der angeführten Tabelle sind die Grundergebnisse der Gleichgewichtsdialyseversuche Kaseinsolen in wässerigen Lösungen von Essig-, Milchund Zitronensäuren wiedergegeben angeschtellt. Ausser den Werten M sind auch die berechneten Werte der wahren Säurebindung S wiedergegeben, welche durch Eiweissalzbildung hervorgerufen ist.

$g = 38.8$ g/l; $v_1 = 4.84$ cm³; $v_2 = 5$ cm³

Säure	Säurenkonzentration C_0	ΔC M/l	a_H^I M/l	a_H^{II} M/l	f_H	C_A^I M/l	C_A^{II} M/l	ΔC_s M/l	ΔC_m M/l	M M/g · 10³	S m/g · 10³
Essig-	1.00	0.0127	0.0013	0.0038	0.931	0.0119	0.0041	0.00377	0.00893	0.45	0.27
»	2.00	9.0204	0.0020	0.0065	0.91	0.0232	0.0071	0.00779	0.0126	0.64	0.54
Milch-	0.25	0.0134	0.0032	0.0060	0.912	0.0123	0.0066	0.0027	0.0107	0.54	0.23
»	0.50	0.0187	0.0049	0.0087	0.896	0.0172	0.0097	0.00352	0.0152	0.77	0.30
»	0.90	0.0210	0.0062	0.0107	0.875	0.0211	0.0122	0.00416	0.0168	0.85	0.36
Zitronen-	0.15	0.0113	0.0066	0.0102	0.885	0.0178	0.0115	0.0029	0.0084	0.42	0.27
»	0.52	0.0144	0.129	0.0204	0.846	0.0381	0.0241	0.00648	0.0079	0.40	0.59

Aus dem angeführten experimentellen Material ist ersichtlich, dass die schwachen organischen Säuren tatsächlich von dem Kasein auf zwei verschiedene Arten gebunden werden: erstens geht eine gewöhnliche chemische

Bindung der Säure durch die freien NH_2-Gruppen des Eiweisses vor sich; zweitens findet eine Aufnahme der untersuchten Säuren durch das Kasein statt. Diese Aufnahme ist mit der Salzbildung von Protein nicht verbunden. Früher wurde diese Sorption organischer Säuren durch das Eiweiss nicht in Betracht gezogen.

Der Mechanismus einer derartigen Bindung der Säure kann gegenwärtig nicht als endgültig geklärt betrachtet werden. Unserer Meinung nach geht in diesem Falle eine Sorption der undissoziierten Moleküle organischer Säuren durch das Kasein vor sich. Diese Sorption steht wahrscheinlich in Verbindung mit einer starken Quellung und besserer Löslichkeit einer Reihe von Proteinen in den wässerigen Lösungen schwacher organischer Säuren gegenüber den Mineralsäurenlösungen, wenn der Vergleich bei denselben H^+-Ionenaktivitäten vorgenommen wird.

Die im Prinzip richtige Schlussfolgerung von Loeb [4] über die Abschwächung der Verkettungskräfte im Gel bei der Wirkung von schwachdissoziierten Säuren fand in der gegenwärtigen Untersuchung experimentelle Bestätigung. Der peptisierende Einfluss schwacher Säuren besteht darin, dass die Proteinteilchen neben der Bindung der H^+-Ionen die undissoziierten Säurenmoleküle sorbieren, was auch eine intensivere Quellung und bei gewissen Konzentrationen der Säure eine Proteinlösung hervorruft.

Die Bindung undissoziierter Moleküle von Essig-, Milch- und Zitronensäure durch Kasein betrachten wir als Solvatation unseres Eiweisses durch organische Säuren. Für diesen Standpunkt sprechen verschiedene Tatsachen. So ist z. B. die Fähigkeit von Essigsäure bekannt, Kasein aufzulösen und folglich auch zu solvatieren. Letzteres ist gleichfalls in hochkonzentrierter Milchsäure (98%) lösbar. Davon zeugt auch die bessere Löslichkeit von Kasein in wässerigen Lösungen schwacher organischer Säuren im Vergleich zu starken Mineralsäuren.

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GENETICS

HETEROCHROMOSOME IN THE WILD SPINACH

By A. G. ARARATJAN

(Communicated by N. I. Vavilov, Member of the Academy, 8. V. 1939)

Various forms of cultivated spinach *Spinacia oleracea* L. are being studied caryologically since 1911 ([10]). However, neither the first writer who investigated this species, nor those who followed him ([4,13]) succeeded in discovering heterochromosomes even though they were in the quest of them ([2,6,9,11,14]). The same species was also subjected to investigation by me under application of new fixing fluids, the weak and strong chromformalin ([5]); however, no unequal pair of chromosomes could be detected.

Caryological data on the wild species *Spinacia tetrandra* Stev. are found only in the works by Lortz ([6]) who failed to find any unequal chromosome pair in this species. Wild spinach is being caryologically studied by me since 1934. The seeds or rather syncarpia and sprouts (in some cases) were collected in the vicinity of Erevan. I have discovered in the material available a clearly expressed unequal pair of chromosomes; it is well distinguished from the rest of the caryotype of the wild spinach. Nearly all the six pairs of chromosomes differ from one another. Besides, five chromosome pairs are strongly heterobrachial, or even possess «knobs»; the sixth pair is composed of nearly isobrachial chromosomes (designated by Lortz as *A*). This chromosome pair consists of equal components in female individuals whereas in the males it is strikingly unequal. One of the unequal chromosomes of the male plant both in size and in shape is a replica of the homologous chromosome of the female plant; the other also is nearly isobrachial, but its length is about 1.75 that of its partner.

Applying the designation adopted for the so-called sex-chromosomes, the formula of the female pair would be XX, that of the male pair—XY. As the Y-chromosome characteristic of the male plants is larger than X, *S. tetrandra* may be referred to the third group following Darlington's scheme ([1]). Wild spinach is so far the single representative of this group among plants. In *Melandrium album* and in a number of other plants possessing a pair of unequal chromosomes Y is always smaller than X.

It is beyond question that some of dioecious plants contain in their caryotypes a heterochromosome. The heterochromosome occurs in the one sex. In the majority of cases it is present in the male individuals; hence it may be deduced that the assortment of the chromosomes of the unequal pair is correlated with inheritance of sex. However, in certain plants (*Rumex*, *Humulus*) there is a more complex group consisting of 3—4 chromosomes which are also distributed in connexion with inheritance of sex ([3,9]).

Many authors have sought heterochromosomes in various dioecious plants, but it failed to be discovered in any of them. One of these plants is the cultivated spinach. Other plants may be cited as well: white mulberry, American maple, *Trinia Hoffmanni* (investigated by me, $2n=18$), etc. Of

course, one may oppose the view that the heterochromosome is present in these dioecious plants, too, but the difference between it and its partner is so slight as to make it indistinguishable. However, such an argument is unfounded and cannot be proved, inasmuch as all the chromosomes are subordinated to the same principles as the whole plants or their parts, and possess individual differences.

Still more interesting is the fact that certain plants possessing hermaphrodite flowers have an unequal pair or complex of chromosomes, too. It is noteworthy that heterochromosomes were first discovered in a hermaphroditic plant *Galtonia candicans* (7). A similar example is offered by a hermaphroditic species *Rumex pallidus* which has a threefold complex of «sex»chromosomes of the kind present in the dioecious *Rumex acetosa* (3). Thus, the heterochromosome is not peculiar to dioecious plants only and its presence may not be connected with the phenomena of sexinheritance.

The fact deserves attention that from the two very closely related species, as cultivated and wild spinach, only one has the heterochromosome. Besides, the wild spinach studied by Lortz (6) has no heterochromosomes. Therefore even within the limits of the species there may exist caryoraces possessing a heterochromosome and those lacking it.

♀ ♂

Nucleus plate of *Spinacia tetrandra* Stev.; × 3 500.

The same is proved also by the following facts. In many plants there constantly appaer flowers of the opposite sex on the male and female individuals. In some of them there are all intergradations between purely male forms to purely female ones and polyoecious like in *Mercurialis annua*(15). One should admit in these cases that gametes of the opposite sex may be formed also from mother-cells containing «unusual» sex-chromosomes, i. e. chromosomes for some reason cease to determine sex or rather that sex in general is not determined by the chromosomes. In addition, there are data which show that members of the «sex» complex consisting of several chromosomes are sometimes distributed in meiosis in various combinations (12).

These facts point to that the presence of the «sex»-chromosome is not obligatory in dioecious plants; it does not determine sex, and if in certain plants there is some kind of connexion between it and inheritance of sex, this connexion is undoubtedly of secondary origin.

The question as to the causes of the formation of heterochromosomes and the character of the connexion between it and sex-inheritance cannot be answered before an adequate study of the phenomena of heredity is made on the basis of all the cellular and extra-cellular processes taking place in the organism, not on the basis of a mere chromosome behaviour.

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CENETICS

RÔLE OF AUTOPOLYPLOIDY IN THE ORIGIN OF SIBERIAN RASPBERRY

By M. A. ROZANOVA

(Communicated by N. I. Vavilov, Member of the Academy, 2. V. 1939)

A morphological and caryological study of the conspecies *Rubus idaeus* L. has made it possible to revise its taxonomic subdivision and to form a new concept of the phylogeny of the intraspecific units. From this study we found that the Siberian race of raspberries which taxonomists ([1],[3]) have referred to *Rubus idaeus* subsp. *melanolasius* Focke, the Sakhalin race, classified as *R. idaeus* subsp. *sachalinensis* Léveillé, the Kamchatka race, named by Komarov ([2]) subsp. *sibiricus* and possibly also the north-western American race (which Focke himself named subsp. *melanolasius*) are all tetraploids ($2n=28$), while the European race, *R. iadeus* subsp. *vulgatus* Arrhen, and the north-eastern American race, *R. idaeus* subsp. *strigosus*, are diploids.

Taking also into account the similarity of the Siberian, Sakhalin and Kamchatka races as regards morphological characters, which was established by a study of both herbarium and living specimens, we consider it correct to unite all these three races nito one species with the name *Rubus sachalinensis* Léveillé, under which name the Sakhalin race was first described as a separate species ([7]). Possibly, the north-western American race, classified by Focke ([5]) as *R. idaeus* subsp. *melanolasius*, should also be referred to the conspecies *R. sachalinensis*, but since we have not had at our disposal living plants to investigate caryologically, we refrain at present from deciding this question. Hence, under *R. sachalinensis* Léveillé s. l. we now include all Siberian , Far Eastern, Sakhalin and Kamchatka forms characterized by a spreading bush, by the greyish-purple colour of their two-year-old shoots which are covered with numerous simple and glandular prickles, by the anthocyanin tinge of the leaves, by the light red fruits which readily drop off by their comparatively early flowering and fruiting and by their low fertility.

In order to solve the problem as to the origin of the tetraploid species *Rubus sachalinensis*, a study was made of microsporogenesis in different specimens of *R. sachalinensis* from the Sayan Mts. and the vicinity of Perm.

As is well known, in autopolyploid species a lowering of fertility is often connected with irregularities at meiosis. An analysis of meiosis in *R. sachalinensis* showed the usual picture for autopolyploids. At diakinesis quadrivalents are formed, usually to the number of three (Fig. 1). In view of the fact that it was not possible in all cases to determine the exact number of quadrivalents, a count was made of the number of chromosome bodies

in 50 plates at diakinesis, and it was found that the average number of bodies was 10.70±0.23, i. e. close to 11, which would result in case there were 3_{IV} and 8_{II} at diakinesis. This indicates that the genoms of this tetraploid are homologous as regards at least three chromosomes. Müntzing ([8]), having made a study of meiosis in tetraploid *Dactylis glomerata*, also found an average of 3.48 quadrivalents, indicating the incomplete conjugation of the chromosomes of homologous genoms. Evidently, during the course of evolution the homologous genoms of autopolyploids diverge somewhat and become not fully homologous.

In the later stages of meiosis in *R. sachalinensis* there are observed considerable irregularities. At I metaphase, as well as at diakinesis, there is frequently encountered the close union of two pairs of bivalents, which indicates that we actually have to do with quadrivalents (also usually to the number of 2 or 3). At I anaphase either all the chromosomes are scattered over the spindle or there is observed a lagging of a few chromosomes. Normal division occurs very rarely. At late anaphase 14 chromosomes are found at each pole; other numbers were not observed. Thus, despite the irregularities at meiosis, 14 chromosomes pass to each pole. At II metaphase 14 chromosomes are also found. The separation of the chromosomes to the poles takes place with less irregularities than at the first division. There were no instances of all the chromosomes being scattered over the spindle, only of the lagging of a few chromosomes. Tetrads are usually formed normally; only occasionally triads or pentads are encountered, ordinarily with normal-appearing cells, but in some cases with degenerating nuclei. The percentage of abnormal pollen in the various races ranges from 3 to 32, i. e. is comparatively small. In view of the fact that the fertility of *R. sachalinensis* is quite low, both under natural and experimental conditions, it is to be presumed that irregularities at meiosis during macrosporogenesis are even more marked.

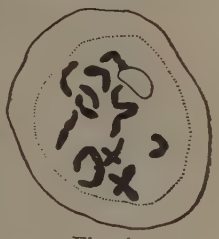

Fig. 1.

The homology of the genoms of *Rubus sachalinensis* has also been confirmed by a study of meiosis in hybrids obtained by us between *R. idaeus* subsp. *vulgatus* Arrhen. and *R. sachalinensis*. These hybrids are completely sterile; their somatic chromosome number is 21.

At diakinesis of these hybrids there are observed trivalents to the number of 2 or 3 (Fig. 2). By chromosome counts in 50 plates at diakinesis it was found that the average number of chromosome bodies is 10.22±0.19. In the case of 10 bodies, it is most probable that there are $4_{III}+3_{II}+3_{I}$, and in the case of 11 bodies —$3_{III}+4_{II}+4_{I}$, and these, in fact, are the combinations most frequently observed in good plates. The smallest number of bodies observed was 8, which might occur in case of $6_{III}+1_{II}+1_{I}$. At I metaphase the number of bodies ranges from 7 to 11. In case of 7 bodies, judging by their size we presumably have 7 trivalents. Hence, on the basis of a study of diakinesis and metaphase in these hybrids, it may be concluded that the number of homologous chromosomes may amout to 7, i. e. that the genoms of *Rubus sachalinensis* were at one time apparently homologous.

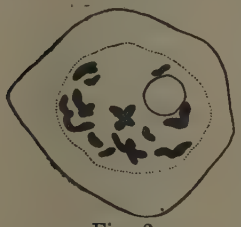

Fig. 2.

Without describing in detail the further stages of meiosis in this hybrid, since they do not furnish any new data on the problem under study, we shall merely point out that both the first and the second division are characterized by marked irregularities. As a result, the tetrads formed comprise, as a rule,

cells of unequal size; sometimes there are also observed dyads, triads, and pentads. The pollen is 95—98% abnormal.

Presuming on the basis of our study of its meiosis that *R. sachalinensis* is an autopolyploid, we believe that it arose from the type *R. strigosus* Mchx. which it resembles in morphological characters.

On the basis of a study of the geographical distribution of polyploids and, in particular, on the basis of a study of the phenomenon of polyploidy in mountain regions (4), we are of the opinion that *R. sachalinensis* may have arisen under mountain conditions in eastern Asia or north-western America.

Hence, the relationship between species belonging to the conspecies *Rubus idaeus* L. may be represented diagramatically as follows:

Initial type *Rubus idaeus* L. → *R. vulgatus* Arrhen.

Initial type *Rubus idaeus* L. → *R. strigosus* Mchx. → *R. sachalinensis* Léveillé.

R. vulgatus and *R. strigosus*, by reason of their morphological peculiarities and their specific geographical areas, may be regarded, just as *R. sachalinensis*, as independent species within the conspecies *Rubus idaeus* and not merely as subspecies as Focke (6) considered them.

The isolation of these species is very marked. But the reason why *R. vulgatus* and *R. strigosus* do not cross with each other is that they are adapted to entirely different climatic conditions, while they do not cross with *R. sachalinensis* because of a difference in chromosome number, thanks to which the progeny obtained from hybridization between these species is inviable.

The establishment of the autopolyploid origin of the Siberian raspberry is not only of theoretical, but also of practical interest as it facilitates the solution of several breeding problems. In Siberia the breeding of small fruits is based on the use as initial material of local forms characterized by their adaptability to Siberian climatic conditions. Consequently, plant-breeding stations in Siberia often take *R. sachalinensis* as initial material. These 28-chromosome forms, when crossed with European 14-chromosome varieties, give sterile or almost sterile hybrids, which is understandable after the analysis of our triploid hybrid. Hence, in order to solve the problem of breeding raspberries for cultivation in Siberia, it is necessary: first, to conduct experiments on crossing *R. sachalinensis* with 28-chromosome varieties; second, to carry out work on crossing north-western Siberian forms belonging to *R. vulgatus* with 14-chromosome European varieties; and, third, to select the 14-chromosome European and American varieties best adapted to Siberian conditions and carry on hybridization work with them.

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GENETICS

INTERSPECIFIC HYBRIDIZATION IN THE GENUS *CUCURBITA*

By K. I. PANGALO and M. K. GOLDHAUSEN

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The species *Cucurbita pepo* L., *C. moschata* Duch. and *C. maxima* Duch. always served as one of the striking examples of botanical species that could not be intercrossed. Four years later, in 1929, there appeared a paper by two American investigators, Erwin and Haber, reporting successful crosses between *Cucurbita* species. There followed a number of investigations by various workers—Castetter, Li, Van-Eseltin, Dana, Lyashchenko—with more or less successful results in the synthesis of such hybrids.

Hence, beginning in 1935 the Institute of Plant Industry included in its research program investigations on interspecific hybridization in the genus *Cucurbita*. The tasks were to obtain, in so far as it might be possible, all hybrid combinations between the four cultivated species—*Cucurbita pepo* L., *C. maxima* Duch., *C. moschata* Duch., and a new species, *C. mixta* Pang.—and to elucidate the phenomena occurring in F_2.

In getting the hybrids it was desirable to avoid making a large number of crosses. Therefore, they were made nearer to autumn, all flowers on the parental plants having been plucked off from the beginning of flowering to the time of crossing. This method proved, apparently, not very effective, since, having made at the end of the summer about 127 crosses, we obtained only 15 fruits with seeds for the eight combinations between the four parental species. Erwin and Haber from 2917 crosses obtained 332 fruits for six hybrid combinations between three species.

All data on the results of our crosses are summarized in the table.

The species under experiment do not cross readily with one another; as a rule, only a few seeds are obtained, and their viability is impaired. The F_1 grown in 1936 showed that the 1935 seeds from most of the crosses were actually hybrid and not apogamous or parthenogenetic. Only in a few cases, e. g. hybrids Nos. 699 and 704, did the F_1 plants arouse doubt, since they were exact replicas of the maternal forms; in one case, hybrid No 700, the F_1 plants were exact replicas of the paternal plant.

Different characters behaved differently: some, such as colour of fruits, texture of flesh and shape of stem, were usually intermediate; others, such as shape and aeration of leaves, were dominant.

Exact reciprocal crosses (i. e. between one and the same varieties) were not made, but it was noted that the characters of *C. moschata*, when crossed with *C. pepo* or *C. maxima*, dominate in F_1, regardless of whether this species is used as the maternal or paternal form. Hence, it is impossible as yet to speak about phenomena of matroclinal or patroclinal inheritance. As to the significance of the variety used in species crosses, there seem to be positive data. Two crosses, the mother plants being two different varieties of *C. mo-*

Hybrid No.	Pedigree	No. of crosses	No. of fruit obtained	Successful crosses (%)	No. of hybrid seeds obtained	Germinating capacity of hybrid seeds (%)	Germinating capacity of seeds from F_1 (%)
701	*C. pepo* No. 1018 × *C. mixta* No. 1709 . . .	2	1	50	207	98	68
699	*C. pepo* No. 1815 × *C. mixta* No. 1188 . . .	3	1	33	56	73	34
700	*C. pepo* No. 1815 × *C. moschata* No. 366 . .	28	2	7	10	10	0
696	*C. pepo* No. 1815 × *C. moschata* No. 571 . .	17	2	12	76	26	56
691	*C. moschata* No. 1657 × × *C. pepo* No. 1748 .	4	1	25	4	75	36
693	*C. moschata* No. 1657 × × *C. mixta* No. 1709	6	2	33	21	95	51
694	*C. moschata* No. 571 × × *C. mixta* No. 1708 .	6	2	33	16	56	74
692	*C. moschata* No. 571 × × *C. mixta* No.1687	30	1	3	35	57	0
703	*C. maxima* No. 1673 × ×*C. moschata* No 571	15	2	12	120	32	37
702	*C. maxima* No. 1687 × × *C. moschata* No. 1664	10	1	10	17	70	16
704	*C. maxima* No. 1662 × × *C. mixta* No. 1709 .	6	1	16	246	70	81

schata and the father plants the same variety of *C. mixta*, gave different F_1. In one case the characters of the vegetative parts of the plant and of the exterior of the fruits were maternal, while the colour of the flesh and the length of the seeds were intermediate; in the other only the characters of the vegetative parts resembled the species *C. moschata*, the shape of fruits was intermediate, and their colour, as well as characters of the flesh, placenta, and seeds were like *C. mixta*. At the same time, however, it should be noted that several characters have a fully definite and like behaviour, no matter from which species the hybrids inherit such characters. For instance, thick and woody rind is always dominant, regardless of whether it originates from *C. pepo*, *C. myxta*, or *C. maxima*; the same holds true as regards coarseness of leaves and pubescence.

The F_1 of all hybrid combinations, except Nos. 700 and 692, gave by self-pollination viable seeds capable of germination (table). From hybrids Nos. 700 and 692 seed-bearing fruits were obtained only by backcrossing to the paternal species *C. maxima*, and then with difficulty. The seeds in these fruits were very few in number and their germinating capacity proved to be very low.

The F_2 was grown in 1937, and it presented a picture of motley and diverse behaviour as is generally characteristic of distant hybrids. There were all gradations, beginning with almost complete absence of segregation and normal viability of plants and ending with extremely varied segregation, including forms exceeding the parental limits and forms with impaired viability. Complex block inheritance was not observed. The independent inheritance of characters was very definite, sometimes even as regards the most minor characters. For example, in the case of several hybrids among the F_2 segregates may be seen forms having typical *C. pepo* leaf blades with *C. moschata*

pubescence, *C. moschata* fruits with *C. maxima* stems. *C. pepo* flowers with *C. moschata* calyx, *C. maxima* flowers with *C. pepo* stamen column, etc.

A study of the F_2 has been made only in a preliminary way with a comparatively small number of plants (150—250) of each hybrid. A caryological analysis has not yet been made.All the hybrid combinations studied may be divided, according to the character of segregation in F_2, into four main groups:

1) Hybrids showing very slight segregation for a few characters—No. 704.

2) Hybrids with a segregation similar to (1) but with a few new forms—No. 701.

3) Hybrids showing extensive segregation as regards all characters, there being forms exceeding the parental limits, forms exhibiting various deformaties, new forms, monstrosities, and forms with impaired viability—Nos. 703, 693, 691 and 696.

4) Hybrids giving an F_2 inviable when self-pollinated; these hybrids may be maintained only by backcrosses, and even then they are deformed and but slightly viable—Nos. 700 and 692.

Among the peculiarities of segregation in various F_2 many interesting phenomena may be noted.

The rate of vegetative growth in most F_2 plants of all hybrid combinations was normal, but in some cases there was observed either an acceleration or a retardation in the growth of the vines. Thus, in the case of hybrid No. 693 by the first of July the length of the main vine of the parental forms was 50—70 cm, while in some F_2 forms it was 175—200 cm. In another hybrid combination, the parental forms showing normal growth, some F_2 plants after having very slowly formed at the beginning 2—3 leaves stopped growth at this stage and during the following 5 months suceeded in making an increase in length of vine of only 3—5 cm. As regards secondary vines, there were very sharp deviations from the parental norms: in three F_2 plants of hybrid No. 694, having a main vine 1.25—1.75 m long and internodes 1.5—2 cm long instead of the usual 5—7 cm, there developed such a mass of vines of the first, second and third order, that they formed an entirely unusual type of bush for *Cucurbita*, similar to a very thick gigantic brush. In contrast to this, a number of F_2 plants of hybrid No. 696 had but one main stem with two very short vines of the first order.

With respect to leaf size, it should be noted that some hybrids far exceeded the parental forms. The latter had leaves averaging 14×16 cm in size, while some F_2 plants had leaves 22×25 cm. Again, mention should be made of the occurrence of some plants with broad, reniform leaves in the F_2 of hybrid No. 701, although the parental forms had cordate leaves with distinct lobes, of the occurrence of aeration of leaves in hybrid No. 693, the parents of which did not show the slightest trace of aeration, of the occurrence of unique (unknown among *Cucurbita*) «pleated» leaves, as if the leaf blades were gathered in folds or pleats, and, lastly, of one plant, in the F_2 of hydrid No. 702, markedly retarded in development and having, instead of leaves, some kind of small out-growths like stunted leaf-stalks.

Most of the F_2 plants developed without special abnormalities and attained the stage of flowering and setting of fruits. In flowering, as in rate of growth, there could be noted either acceleration or retardation. Thus, in hybrid No. 696 by the middle of July the parental forms had not flowered, while all the F_2 plants were covered with male blossoms and 50% of them also had female blossoms. In hybrid No. 693, on the other hand, the parental forms were in full bloom, while not even buds had yet made their appearance on the F_2 plants.

Especially great changes were observed in F_2 flowers. As regards size, the limits set by the parental forms were exceeded in both directions. For instance, in hybrid Nos. 701. 696, 694 and 702 there were plants whose

female blossoms averaged only 3.2 cm in diameter, while those of the parental forms were 6 cm in diameter. In other hybrids the reverse held true.

In a number of hybrid combinations there occurred segregates with flowers entirely or almost entirely fasciated (peduncles, stamen column, corolla, etc.). In hybrid Nos. 692 and 693 there were segregates having flowers without any perianth. In hybrid No. 693, where the parents had a distinctly inferior ovary, there were F_2 plants with a half-inferior ovary, as in Turban squanshes. In the F_2 of hybrid No. 691 there were plants having apocarpous pistils with ovules along the edges; the latter developed and attained the size of flax seeds, and then withered away. From many of the hybrid combinations there segregated out plants with androgenic-hermaphrodite and gynogenic-hermaphrodite pistillate flowers varying in the degree of development of hermaphroditism. In several forms pure female flowers did not occur at all during a prolonged period; in place of them were hermaphrodite flowers.

As regards fruits and fruiting, there were less peculiarities than with respect to flowers and flowering. The most interesting was the appearance in the case of two hybrid combinations (Nos. 696 and 691) of forms maturing their fruits earlier or later than the parental forms. The latter matured their fruit at the beginning of September (*C. pepo*) or at the end of September (*C. moschata*), while some F_2 plants from the first combination (hybrid No. 696) gave ripe fruits by the end of July and some from the second combination (hybrid No. 691) did not produce any mature fruit even in October. As regards number of fruits, there were interesting cases of forms exceeding the limits set by the parents. Thus, in the case of hybrid No. 696 there were segregates bearing as many as 40 fruits per plant, whereas the parent bearing the most fruits, a small-fruited ornamental form of *C. pepo*, bore a maximum of 15 fruits per plant. With respect to fruit size, there were no segregates with fruits larger than those of the parental forms, but segregates with smaller fruits occurred in the case of hybrid No. 699, some bearing fruits only half as large as those of the smaller-fruited parent.

In addition to the enumerated peculiarities of these interspecific *Cucurbita* hybrids, it is necessary to note the occurrence of diseased plants, deprived without apparent cause of the ability to develop and remaining during the entire vegetative season at the stage of seedlings with only 3—4 leaves; of plants having leaves with yellow spots and developing abnormally; of dwarf, deformed bushes of insignificant size, having, instead of leaves, small outgrowths like stunted leaf-stalks (such forms did not flower, but lived all summer); and, lastly, of fully viable and symmetrical dwarfs, only one-half or one-third as large as the parental forms but flowering and fruiting normally.

The results obtained from this study of interspecific *Cucurbita* hybrids have revealed a number of interesting facts, and indicate that many problems of experimental evolution may be cleared up by further hybridization work. At present it is possible to draw only a few conclusions as to the mutual affinity of *Cucurbita* species. Most closely related to each other are *C. moschata* and *C. mixta*; further apart stands *C. pepo*; and quite distant from all the other three species stands *C. maxima*.

After these preliminary experiments the Institute of Plant Industry is undertaking a thorough study of species hybrids in *Cucurbita* from a genetical and plant-breeding standpoint.

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APPARATUS FOR GASOMETRIC DETERMINATION OF CATALASE IN PLANT TISSUES

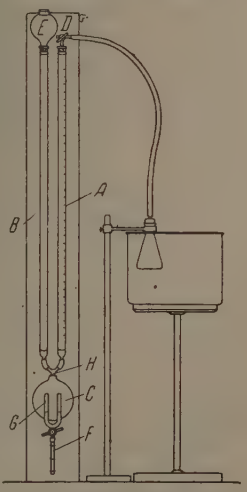
By V. S. SHARDAKOV

(Communicated by N. I. Vavilov, Member of the Academy, 8. V. 1939)

Determination of the catalase activity by gasometric method is usually done either in the eudiometer (1) or in the apparatus of Heinicke-Hott. The apparatus of Lishkevich (2) as described by N. N. Ivanov (3,4) is widely used. However, the latter has a material drawback which consists in the system of gas pressure control, the levelling of the gas pressure with that of the atmosphere being done in the closed portion of the apparatus by hand, through letting down a glass squirt containing the reserve liquid. It is rather difficult to make such levelling with an appreciable degree of exactitude when pressure is changing rapidly under the influence of oxygen liberation. The great diameter of the squirt as compared to that of the graduated tube on which the reading is taken makes exact measurements still more difficult.

This defect is eliminated in the apparatus recommended here. It is essentially Warburg's microphotometer but somewhat modified and considerably enlarged.

The apparatus is simple and may be assembled at any laboratory or ordered at a glass-blower's. It is diagrammatically shown in Fig. 1. It consists of two glass tubes *A* and *B* of equal diameter connected at their lower ends by a V-shaped three-way glass pipe *T*. One of them *A* is divided into tenths of a cubic centimeter. Together they constitute the manometric part of the apparatus. An ordinary burette of 25 cm³ capacity without a valve may serve as graduated tube which is made of an ordinary glass tube having the same diameter as the burette. These tubes are connected by means of a rubber-pipe and a three-way pipe; the lower branch of the latter is fixed in a rubber squirt *C* of 150—200 cm³ capacity (from a medical syringe). Into the upper end of the graduated tube (burette) a three-way pipe *D* is inserted (by means of rubber): it is supplied with a three-way valve. Two ends of this three-way pipe, the one inserted into the burette and the other leading into the open air, are cut off short; the third one leading into the reaction retort is left longer. To avoid liquid being ejected from the left non-graduated tube, a glass squirt *E* (from a drop-funnel) is inserted into its upper part by means of a rubber stopper or a rubber pipe.



Apparatus for gasometric determination of catalase.

All these parts of the apparatus are mounted on a wooden board. Fasten-

ing is done with wire loops. The tubes are arranged parallel, as close to each other as possible. To level the pressure within the apparatus by changing the volume of the rubber squirt containing the reserve manometric liquid, a screw-clamp *F* commonly used to hold Liebich's condensers is fastened to the board. It embraces *G*, the squirt, so that one can change the level of the fluid in the two tubes of the manometer by turning the clamp's screw.

Stained water is used as manometer liquid. The apparatus is filled up with water through the aperture of the glass squirt *E* of the left non-graduated tube. Water is poured in a quantity sufficient to raise its level somewhat above the zero line of the graduated tube when the squirt is clamped. No air should be left within the squirt. The long arm of the three-way pipe *D* is connected by means of a long rubber pipe with a glass tube passing through a rubber stopper of an Erlenmeier's retort of 150 cm^3 capacity. The retort is placed, as usual, on a thermostatic water-bath and is hold by a stand with a spring-clamp. The retort must be fixed rather loosely so that one could shake it.

Determination of the activity of catalase with the aid of the apparatus described is done in the following way. A definite amount of material is ground down following the generally adopted technique ([3]). The resulting pulp is placed into an Erlenmeier's retort. Then a vial containing hydrogen peroxide is placed into the retort. The retort is connected with the apparatus by means of the rubber pipe and placed into the thermostatic bath. The three-way valve of the three-way pipe of the graduated tube connects at this time the apparatus and the retort with the atmosphere. Upon levelling of the temperature the level of the liquid is brought to the zero line of the graduated tube of the manometer by changing the volume of the rubber squirt with the aid of the screw-clamp. The valve of the apparatus is then disconnected from the outside air and connected with the retort. After having taken the reading of time by means of a stop-watch, the vial containing hydrogen peroxide is turned over by shaking the retort. Liberation of oxygen due to disintegration of the hydrogen peroxide under the influence of the ferment raises the pressure in the closed arm of the manometer so as to make sink the level of the liquid.

After a due period of time, the manometric tube is closed and disconnected from the retort with the aid of the three-way valve of the three-way pipe. The retort at this moment becomes connected with the outside atmosphere. The liquid in the two tubes of the manometer is brought to a common level by turning the clamp's screw. A reading which characterizes the relative activity of the ferment is made on the graduated tube.

When investigation of the dynamics of the activity of the ferment in the sequence of time is needed, one should operate in the following manner: the apparatus ready to use is disconnected from the atmosphere and the reaction started. Without changing the position of the valve, the liquid is kept at the same level in the two tubes by turning the screw of the clamp compressing the rubber squirt which is gradually unclasped; readings are made on the graduated tube at desired moments. The screw-clamp makes it possible to keep easily and conveniently the liquid at the same level in the two tubes.

This apparatus proved to be considerably more precise and convenient than the common ones; this is especially true of mass determinations.

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A SIMPLIFIED PROCEDURE OF MEASURING LEAF AREA BY MEANS OF A PLANIMETER

By V. S. SHARDAKOV

(Communicated by N. I. Vavilov, Member of the Academy, 8. V. 1939)

Contact copies of the leaves on sensitive paper are usually resorted to in determining the leaf area with the aid of a planimeter. The outlines of the leaves on the paper copies are then followed up by the planimeter.

Below is described a very simple device which makes it possible to avoid the time-consuming and tedious work of preparation of the copies. The result is attained by directly contouring with the planimeter the leaf placed under glass. To preclude sliding of the working wheel of the planimeter (sliding should be avoided in any event) and to bring it in one level with the contour of the leaf, the latter is put under a large glass. This glass is papered (drawing paper is used), the area under which the leaf is placed being left free of paper. The base of the planimeter which is steadily fixed is placed beside the glass on the desk. Although the base-point is thus at another level, the ball-link connecting it with the movable carriage of the planimeter prevents any damage to the exactitude of measurements.

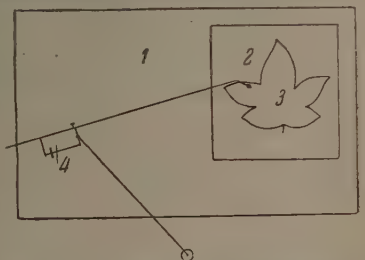

Fig. 1. *1*—glass, *2*—rectangular aperture, *3*—leaf, *4*—planimeter.

The size of the glass should be sufficient to give room to the working wheel of the planimeter while contouring the leaf. The area of the glass not covered with paper depends upon the size of the leaves to be measured. Trials have shown, for example, the following sizes to be usable: 35×20 cm glass, the free area 12×15 cm being placed in the upper right corner of it.

To reduce the parallactic error as much as possible, the glass should be thin. However, this error is not great since, although the visible contour is shifted with regard to the real one, the two remain parallel to each other.

The construction of the device and the relative position of the planimeter may be seen from the accompanying diagram (Fig. 1).

Trial of the simplified method recommended here was made in the following way. Areas of ten leaves of apple-tree were first measured by the usual way with the aid of a planimeter. Contouring of each leaf was done four times. Then contouring was done through glass as described above. Each of them

was also made four times. The mean values obtained from the four determinations of the leaf area were then compared to one another.

The results are summarized in the table.

Leaf No.	Area of leaf of apple.-tree in dm^2		Difference in the leaf areas in dm^2
	Obtained directly by contouring with the planimeter	Obtained with the aid of the device described	
1	0.2719	0.2720	— 0.0001
2	0.4335	0.4335	0.0000
3	0.2081	0.2060	+ 0.0021
4	0.1575	0.1598	— 0.0023
5	0.1905	0.1847	+ 0.0058
6	0.1989	0.2019	— 0.0030
7	0.3883	0.3830	+ 0.0053
8	0.3280	0.3260	+ 0.0020
9	0.3650	0.3640	+ 0.0010
10	0.1630	0.1600	+ 0.0030
	2.6747	2.6609	+ 0.0142

From the table it is evident that the differences in area determined by the two methods do not exceed 0.006 dm^2 (0.6 cm^2) for individual leaves. For the total area of the ten leaves the difference equals 0.0142 dm^2 (1.42 cm^2). The relative difference for the latter case is 0.54%, i.e. rather negligible, so that the use of the method may be recommended.

Trials have amply demonstrated the fitness of the device and the considerable saving of labour and time as compared with the ordinary method.

The leaves collected for determination of the areas should be kept in a damp atmosphere.

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ECO-GEOGRAPHICAL CHARACTERISTIC OF THE WILD SPECIES OF BEET (*BETA* L.)

By V. P. ZOSSIMOVICH

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Phyto-geographical investigation on wild and cultivated beets in the USSR was followed by an anatomical and caryological study. Interspecific and distant interracial hybridization in wild and cultivated (*B. vulgaris*) beets was conducted. The results of genetical and taxonomic studies made it possible to undertake a revision of the modern ideas of the taxonomy of the genus *Beta* L. and to obtain a number of hybrids some of which may be soon turned to practice.

This study envolved various materials gathered by means of expeditions. Experimental studies have demonstrated that the genus *Beta* L. has split during its evolution into three natural sections (1) presented in Table 1.

Table 1

Sections and species	Date of its discovery	Diploid number of chromosomes	Geographic area of the species
I. Sect. *Corollinae* Transh.	(1927)	—	—
Beta lomatogona F. et M.	(1838)	$2n = 18$	Turkish Anatolia, Iran, Transcaucasia
Beta nana Boiss. et Held	(1846)	unknown (18?)	Greece
Beta intermedia Bunge	(1879)	unknown (36?)	Central Turkish Anatolia
Beta trigyna W. et K	(1802)	$2n = 54$	Western Anatolia, Balkan Peninsula Crimea
Beta corolliflora Zoss.	(1937)	$2n = 36$	Transcaucasia, Iran, North-Western Anatolia
Beta macrorhiza Stev.	(1812)	$2n = 18$	Eastern Turkish Anatolia, Iran, Transcaucasia
II. Sect. *Vulgares* Transch.	(1927)	—	—
Beta vulgaris L.	(1753)	$2n = 18$	India, Asia Anterior, Mediterranean region, Atlantic Coast
III. Sect. *Patellares* Transch.	(1927)	—	—
Beta patellaris Moq.	(1849)	$2n = 18$	North-Western Africa, Southern Spain, Canary Islands
Beta procumbens Chr. Sm.	(1815)	$2n = 18$	Canary Islands, Cape Verde Islands
Beta Webbiana Moq.	(1840)	unknown	Canary Islands

It is very probable that the oldest of these sections is sect. *Vulgares* which has segregated out the section *Corollinae* in Asia Anterior and the sect. *Patellares* in the Western part of the Mediterranean region and at the Canary Islands. We suppose that sect. *Vulgares* is linked with sect. *Corollinae* through the species *B. macrorhiza*, and with sect. *Patellares* through species *B. patellaris*.

The basic province of speciation of the genus *Beta* L. is in Asia Anterior. In these geographic regions the process of species formation within the genus *Beta* L. has developed most widely. The second region of formation of species of *Beta* L. is in the Canary Islands.

Caryological studies ([3,4]) have shown that the chromosome number remained on the whole unchanged throughout the evolution, being equal to eighteen. The group *Corollinae* is characterized by longest chromosomes and by polyploids. Sect. *Vulgares* is characterized by shorter chromosomes. A general rule was discovered following which the ratio of limbs of homologous chromosomes is very similar in the basic genom of all the species studied. The sterility of most of the interspecific hybrids with equal chromosome number points to that the divergence of the *Beta* species depends aside from eco-geographical isolation on intrachromosomal alterations, too.

The species of the sect. *Corollinae* constitute a common climatype of winter cold-resistant perennial plants. The root in all the species is cylindrically-spindle shaped, often dichotomically branched and penetrates into the soil one meter and more deep. In its upper part root merges into «digits» which originate from the stalk; they bear rosettes of leaves and, later on, stems. It was established ([5]) that the root in *Corollinae* is characterized by significant development of mechanical tissues which lie between vascular bundles (with the exception of *B. macrorhiza*). At an age of several years the root in *Corollinae* attains a weight of 5—10 kg ([2]). Sugar content and dry substance of the sap in *Corollinae* may be characterized toward the end of the growing period of the first year of life by the following values (Table 2).

Table 2

	Sugar, %				Dry substance in the sap, %			
	Mean		Min.	Max.	Mean		Min.	Max.
	M	*V*			*M*	*V*		
Beta macrorhiza	9.76	15.6	5.4	15.2	20.64	6.83	18.0	24.5
Beta lomatogona	7.79	19.5	3.4	11.2	16.25	11.08	11.0	21.5
Beta corolliflora	11.71	7.6	9.6	14.1	23.3	3.69	21.0	25.0
Beta trigyna	11.62	11.9	8.8	15.4	20.05	11.3	15.0	28.0

The root in *Corollinae* is not very productive in the first year of life. The ratio between sugar and dry substance in the sap differs from that which has been created by man by selection in sugar beet. The species *B. trigyna* ($2n=54$) is a natural amphidiploid ([6]); it has originated from a cross between two pre-existing species, *B. lomatogona* F. et M ($2n=18$) $\times$ *B. corolliflora* Zoss. ($2n=36$). The species *B. corolliflora* Zoss. ($2n=36$) distinguished by us was formerly considered to be an assemblage of forms of *B. trigyna* growing in Transcaucasia, in Western Iran and in the northern part of Eastern and Central Turkish Anatolia. The tetraploid species *B. corolliflora* is characterized by largest stems and by a number of characters which distinguish it from other species. The species *B. lomatogona* is a drought resistant ecotype, whereas the species *B. corolliflora* and *B. macrorhiza* are hydrophile ecotypes of perennial species. *B. intermedia* and *B. trigyna* assume an interme-

diate position. *B. nana* is a typical high-mountain dwarf-plant. *B. lomatogona* and *B. nana* are characterized by single-seeded fruits. The significant polymorphism in *B. corolliflora* is in good accord with its possible tetraploid origin. It is very probable that this species had originated from regeneration of calluses in the «digits» of stalk-origin. Hybrids *B. trigyna* ($n=27$)$\times$ $\times$*B. lomatogona* ($n=9$) are fertile and are phenotypically very close to *B. intermedia* Bunge. It is quite possible that the species *B. intermedia* is a natural hybrid of the ordinary, not amphidiploid type.

The species *B. vulgaris* L. is the only authentic species which represents the sect. *Vulgares*. As the native country of *B. vulgaris* should be considered the saline steppes of Asia Anterior; from there its populations, moving eastward, reached India. Moving westward, through the Mediterranean region populations of *B. vulgaris* reached the Canary Islands in the south and Sweden in the north. In connexion with various ecological conditions of its vast area, the wild beet, *B. vulgaris* ssp. *maritima* L., may be subdivided into the following three climatypes:

1. Southern drought-resistant of low sugar content, early maturing wild beet of the «summer» type.
2. Southern hydrophile climatype also of the «summer» type.
3. Northern hydrophile of high sugar content, cold-resistant climatype of the wild beet of «winter» type.

The southern drought-resistant climatype ssp. *maritima* L. is represented by the following branches:

a) proles *syrio-messopotamicum*—Syria, Messopotamia; b) proles *irano-transcaucasicum*—Iran, Transcaucasia; c) proles *mediterraneum*—Mediterranean province.

To the same climatype belong the subspecies *B. vulgaris* L. ssp. *Macrocarpa* Guss.—Mediterranean and ssp. *patula* Ait—western part of the Mediterranean, Island Madeira.

To the southern hydrophile climatype belong the following branches:

d) proles *orientalis*—Northern India; e) proles *italo-adriaticum*—coast of the Northern Adriatic and Italy.

To the northern hydrophile climatype belongs the branch *B. vulgaris* ssp. *maritima* L.; f) proles *atlantico-europeum*—coast of Ireland, England, Northern France, Belgium, Denmark, Holland and Sweden.

All the representatives of the southern climatype bloom here in the first year of life. All the beets of the northern climatype form rosettes in the first year and bloom on the second year of life. This wild beet possesses an enhanced resistance toward the cercospora. A number of characters of the southern wild beet are dominant to the contrasting characters of the wild beet of the northern climatype.

In Table 3 are shown differences in the weight of the root, in the sugar content, dry substance of the sap in the wild beet ssp. *maritima* L. of the northern and southern climatypes.

Table 3

Origin of the population	Weight of the root *M*	Sugar, %			Dry substance (refractometer), % *M*	Number of rings of vascular bundles
		Mean *M*	Min.	Max.		
Southern climatype — Algeria	17.3	4.8	0.2	10.0	10.1	4.2
Northern climatype — England, Belgium, Denmark	230.0	15.3	4.8	20.4	21.6	5—12

The wild beet of the southern climatype develops woody roots containing mechanical tissues; in the northern climatype the latter are much scarcer (9). The chromosome number in wild beet ssp. *maritima* is equal to 18 (3, 4).

Sugar content of the wild beet of the southern climatype very closely approximates that of the southern cultivated beet of Asia Anterior and of the fodder- and table-varieties of Western Europe.

Sugar content of the wild ssp. *maritima* of the northern atlantic-european climatype is near to some of the forms of the leaf-beet and to sugar-beet. This is the principal difference with respect to sugar content between the wild beet of the northern climatype and that of the southern climatype.

The second province of species formation of wild beet is confined to the Canary Islands. Species of wild beet from the Canary Islands, *B. patellaris* Moq., *B. procumbens* Chr. Sm. and *B. Webbiana* Moq., form a well delimited section.

The species *patellares* belong to the southern climatype. Their sugar content is very low. Roots are woody. The species *B. patellaris* and *B. procumbens* are annual-biennial plants. *B. Webbiana* is a perennial. Hybrids F_1 *B. procumbens* $\times$ *B. vulgaris* are sufficiently fertile (8).

All these species are characterized by single-seeded stony fruits. The chromosome number in *B. patellaris* and *B. procumbens* is equal to 18 ($2n=18$). The species *B. procumbens* is resistant to nematodes (8).

As a result of a study of wild species and forms of beet a number of valuable characters were discovered. Here belong winterness, cold-resistance, large size of roots, drought-resistance, single-sidedness of fruits, increased resistance to cercospora, to nematodes, etc.

At the same time the wild beet is characterized by a number of qualites which are undesirable in a cultivated plant, viz., low position of the root, increased woody structure, branching of the root, increased earliness in southern forms, etc. Therefore, the use of the wild species and forms of beet in breeding work is possible only through hybridization between wild and cultivated beets.

At the present time we dispose of a number of hybrids between the wild and cultivated beet obtained by us. Hybrids ssp. *maritima* L. $\times$ *B. vulgaris* (sugar-beet) (7) are very valuable owing to the fact that as early as in F_3 a number of progenies show good productivity and sugar content; in addition, they are much more resistant to cercospora than sugar-beet is. Interspecific hybrids of the cultivated beet with the species *B. lomatogona*, *B. macrorhiza*, *B. corolliflora* are yet in the state of F_2 seed.

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BOTANY

EVOLUTION OF CULTIVATED BEET *B. VULGARIS* L.

By V. P. ZOSSIMOVICH

(Communicated by N. I. Vavilov, Member of the Academy, 8. V. 1939)

The area of the wild forms of the species *B. vulgaris* L. extends over India, South-Western Asia, Mediterranean region, the coast of North-Western Africa and the coast of Western Europe. The introduction of beet into cultivation might have been initiated by peoples which inhabited this vast area.

The variety of forms of cultivated beet is concentrated in Asia Anterior and in Western Europe, where the initial introduction of beet might most probably have taken place.

In ancient times beet was a garden- and a medicinal plant. Growing vegetables as a branch of agriculture appeared much earlier in South-Western Asia than in Southern or Northern Europe. The local population of Asia Anterior is even now using as salad leaves of wild *B. vulgaris* L. growing as a weed. It may be supposed that the wild beet of the species *B. vulgaris* was introduced into cultivation in the valleys of Tiger and Eufrates from the weeds occurring in irrigated agriculture. Conditions of irrigation made it possible to select from populations of wild beet good leaf-producers and later maturing forms ([11, 12, 13]). We think this to be the way the selection for leaves was started some 4000—3500 years ago.

First information on beet are referred only to the VIII century A. C. Its name in Babylonia was «silqā». Large-rooted beet appeared much later than the leafy one, probably in the VI—V centuries A. C. The contemporary names of root-yielding beet in Asia Anterior are of Iran origin. In Iran it was called «gvundar, gundar, gundur». In Turkey it is called «shugundar», «shagonder», «chugundar». Armenian names for beet are «chekender», the Talysh name is «chagandy», etc.

Babylonian, Chettian and Phoenician cultures undoubtedly had an influence upon the further development of the mediterranian antique culture. The influence of ancient Egypt upon the agriculture of the countries of Southern Europe was less important. It may hardly be considered proved that beet was not cultivated in ancient Egypt.

In the remotest past the principal traffic of the products of plant industry was from the countries of Asia Anterior to the Mediterranean region, not the other way round. One may admit, therefore, that cultivated leaf-beet at the beginning of the first millennium A. C. was spread by Phoenicians from Messopotamia and Syria to their Sicilian and Greek colonies. Then begins its cultivation and amelioration in antique Greece and Rome. Theophrastus ([1]) had already pointed out the leafy character of beets in ancient Greece. The

Greeks distinguished white beet (tevtlon=sevtlon leucon) and black beet (tevtlon melan). The Romans like ancient Greeks distinguished between the white leafy beet *Beta* candida (*alba*) and the black one (*Beta nigra*).

The commentator Barbar (1566) in his comments to the work of Dioscorides ([2]) «Pharmacorum» writes that «true black beet in contrast to the white one grows in the fields of Greece».

«The Creeks prefer beet from Ascra and everywhere the white one and call it Sicilian (Graeci preferunt ex Ascra ubique candidam parcissimi seminis appelantes siculam)». Beet was then cultivated in gardens so that one should understand tevtlon melan (*B. nigra*) as the field wild beet called so for dark-green leaves, not the red root-yielding beet as supposed Lippmann ([8,9,10]).

It is possible that t.melan=*B. nigra* was the name of the leaf-beet natural hybrids between light-green with the wild beet. The word *Beta* is of Roman origin. The sense interpretation of the term *Beta* is the easiest one to admit. The Romans had probably grasped the similarity of a young seedling of beet with the Greek letter β (beta).

The system of cultivated beet of the time of ancient Rome abounded in leaf-yielding forms (ssp. *cicla*) and was lacking root-yielding ones. Cultivation of cereals in Western Europe at the times of Caesar and Tacitus was insignificant. There was no idea of gardening. Probably just about the I century of our era the leaf-beet was spread from Rome to its provinces to the west from Rhein. In Western Europe the leaf-beet became later maturing and more sugary. The process of its hybridization with the local highly sugary wild beet ssp. *maritima* L. was in progress.

When arguing the Asia Anterior origin of a number of forms of leaf-beet of Western Europe, one should note the presence of light-yellow leaves and white petioles. The same characters are peculiar to some of the forms of the southern wild beet *B. vulgaris* L. v. *foliosa* from Asia Anterior. Besides, the morphology of the roots and low sugar content of the light-yellow leaf-beet closely approximate those of the Asia Anterior wild beet.

After the fall of Rome the relations between the Western Europe and the Near East came into decay, and early in the Middle Age only leaf-beet was cultivated there. The names of leaf-beet in early medieval France are clearly of Roman origin (Bete, bette, betta, blette, reparée, iotte, poyrée, poirée). The old-German names of beet consistently point to its leafy character and its Roman origin [cicula, Romsch chrut (Kraut), Romisch Kol (Kohl), mangelchrut, mangelt, mangolt].

All these terms and characteristics applied to the beet show that no root-yielding beet was in existence in Western Europe prior to the Crusades. At the same time in the Eastern Arabian califate root-yielding beet of varied root colours was already under cultivation ([5]). The old Arabian names of beet are «selk, silk», the Spanish name is «acelga», the Portugese—«aselga». The root-yielding beet was cultivated at that time in Transcaucasia, in Iran, Khorossan, Khiva, Bukhara, Fergana and Afganistan. At the same time, Arabian trade had distributed beet to India, China, Japan and Eastern Siberia. Root-yielding beet was widely cultivated in Byzantia where it was called «seskla, sescula, sevtlon, sevclon, sevcla».

The old slavic name is «sveklu», the old Polish—«svikla» and the Lithuanian—«sviklos», the Albanian—«sfekle, sfeklia» and the Hungarian «zekla» are of Greek-Byzantine origin. It may be assumed that beet was introduced into the Kiev Russia from Byzantia in the IX—X centuries.

European merchants took advantage of the Crusades to invade the markets of the Near East. It may be supposed, therefore, that root-yielding beet was imported from there, together with many cultivated plants of the Near

East, and began to spread over Western Europe in the XIII—XIV centuries. Cultivation of the root-yielding beet in Western Europe has begun in Northern Italy and the Rhine regions of Germany which adjoined the trade-route with the Levant.

At the beginning the forms of root-yielding beet which were entirely new to Europe were taken for turnips and were called «Rapum alterum, Roth Rüben». There was a distinct name for the leaf beet: «mangel, mangelt, Romischkol, bette, beete, poirée», etc.

Even in XV and XVI centuries in German «Herbaria» the root-yielding beet was placed among *Cruciferae*. Cultivation of leaf-beet together with root-beet in the gardens of Western Europe inevitably resulted in their natural hybridization. The leaf-beet has shown its high sugar content when crossed with the root-beet poor in sugar. As a result mixed populations were cultivated; they contained pure forms of leaf- and root-beet and their hybrids.

On the border of the XVII century, Olivier de Serres ([3]) gave to the root-yielding beet the name «beet-turnips» (bette-rave) and wrote that it has been brought from Italy and that its sap when boiled resembles sugar syrup. C. Bauhino ([4]) as early as in the XVII century classified beets only by the sizes and by colour of the leaves.

In the XVIII century, in connexion with the progress of agriculture, mixed populations of beet were brought from gardens to fields. Root-yielding and hybrid beet was used in Germany as a fodder plant under the name «Burgunderrüben, Runkelrüben, Dickwurzel» etc.

Leaf-beet (ssp. *cicla*) is cultivated in France and Belgium as a salad plant. A number of forms of root-yielding beet which were imported to France from Germany received the name «racine de disette», instead of «betterave».

As early as in the middle of the XVIII century in Western Europe begins the divergence of the poor in sugar root-yielding beet (ssp. *esculenta* into the fodder- and table-lines. In 1747 Marggraf established the presence of sugar in the beet. Lueder ([6]) wrote that «Runkelrübe» is the beet with yellow roots and that «it has originated from a mixture pollen of the great or Swiss leaf-beet (ssp. *cicla*) with the yellow and red (root-yielding, *V. Z.*) beet».

«Its seeds have, therefore, the appearance of the leaf-beet and red beet». There is here a clear indication of the hybridity of a number of beet populations of that time, owing to natural crossing between the leaf- and root-beet. Only at the end of the XVIII century Achardt ([7]) began the work on gaining sugar of the beet.

He made the first selections of the parental forms of sugar-beet. White Silesian beet was selected by Achardt and Coppy from mixed populations of Runkelrüben, Burgunderrüben; it was undoubtedly a natural hybrid between the leaf- and root-beet. The first populations of the Silesian beet of the time of Achard already possessed increased sugar content (10—12%) as compared with the typical poor in sugar (6—7%) root-beet.

In the origin of the sugar-beet took part not only forms of ssp. *cicla* closely related to ssp. *maritima* from Western Europe which gave allelomorphs of high sugar content, but also a number of forms of southern leaf-beet with white or light-green petioles and with large leaves.

From the white-rooted leaf-beet (ssp. *cicla*) the sugar-beet has inherited a number of characters not peculiar to the root-beet, viz. 1) high sugar content, 2) root structure, 3) increased woodiness of the root as compared to the root-yielding beet, 4) increased number of leaves and lower position of the root, as well as the white colour of the latter. The increased productivity of the root of the sugar-beet as compared to the branched roots of the ssp. *cicla* was inherited by the sugar-beet from the root-beet (ssp. *esculenta* L.).

Sugar-beet was thus derived not directly from the fodder,- leaf- or wild ssp. *maritima* L. It is a product of selection among the products of recombination of characters of two pre-existing forms of cultivated beet.

At the present the process of formation of highly sugary plants of the type of the sugar-beet among the hybrids *cicla* × *esculenta* has been experimentally demonstrated ([14]).

Further selection of sugar-beet from the Silesian one was connected with an accumulation of allelomorphs of high sugar content in populations of sugar-beet and with elimination low-producing plants of the type of leaf-beet or poor in sugar hybrids and of typical root-beet. An especially great number of varieties of fodder- and table-beet were produced in Western Europe during the XIX and XX centuries. Among leaf-beets which is being now forced out by other forms, there is a number of varieties of hybrid origin with the root-beet. Finally, a great many varieties of sugar-beet are cultivated now, while only a few belong to half-sugary beet. Root-beet from Asia Anterior is represented now by mixed segregating populations of poor in sugar drought-resistant beet.

Such may be imagined the evolution of cultivated beet which gave the modern variety of forms; we departed here from the basic ideas of Darwin taking into account both divergence and the principle of common origin.

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TRANSFERENCE OF SUGAR-BEET POLLEN BY AIR CURRENTS IN VERTICAL DIRECTION

By I. V. GERMAN

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Problems of sugar-beet breeding and seed selection cannot be successfully solved without devising adequate methods to control pollination.

Among the various measures which must combine to make possible a controlled pollination of sugar-beet, the isolation of small groups of selected material during the first stages of propagation as well as various combinations during the process of crossing are still unsustained by a reliable technique. True, a number of problems, such as the isolation of particular parts of plants or of particular plants, hybridization after castration, as well as the isolation of groups of plants for the purposes of crossing, may be considered as solved ([1–4,8]).

The problem of isolating large plantations, however, has not been sufficiently worked out. In order to isolate large sugar-beet fields (occupying from 200 to 250 acres of land) it is essential that they should be wide distances apart; this may relatively guarantee against undesirable pollination.

The force and the direction of the wind, the contours of the locality, the presence or absence of natural obstacles to the wind, the presence of various insects that carry the pollen from plant to plant, the size of the fields producing or receiving the pollen—all these are highly important factors in pollination.

Many works have recently been written on the wide subject of pollination by the aid of insects. However, utilization of the wind as an agent of pollination requires close study and experimentation ([6, 7]).

While studying transference of sugar-beet pollen, attention should be paid to the movement of the air, both in horizontal and vertical direction, as well as to the size and form of the pollen grain ([5]).

In 1938 the author of these lines, under the guidance of Prof. A. Z. Arkhimovich, head of the Plant-breeding Chair of the Zhitomir Agricultural Institute, conducted experiments on vertical dissemination of sugar-beet pollen. A sugar-beet plantation of about 280 acres which was in blossom was set apart for this purpose in the Chervonskoye State Farm by the Zhitomir Sugar Trust.

In order to ascertain the presence of pollen grains in atmospheric strata four flights were made in an aeroplane at various periods of blossoming and in different times of the day. The interception of the pollen grains was done by means of numbered glass slides covered with glycerin and gelatin and atta-

Total Amount of Pollen Grains

Flights	Altitude			
	50	100	200	300
1st flight — Total	1173.1	846.2	615.1	651.7
in %	26.39	19.04	13.83	14.66
2nd flight — Total	1501.7	1243.2	1097.5	932.2
in%	23.25	19.26	17.0	14.44
3rd flight — Total	1475.7	1034.2	948.5	845.9
in %	23.94	16.93	15.39	13.71
4th flight — Total	1208.3	800.0	883.6	801.1
in %	23.23	15.38	17.0	15.45

ched to specially designed forks. At the moment of exposure the forks were protruded from the forepart—the seat of the observer. The duration of the exposure of the glass slides at each altitude varied between 8 and 10 min.. the aeroplane flying at a speed of 120 km an hour.

While intercepting pollen at various altitudes care was taken to have a uniform speed developed by the plane. Each flight lasted from one and a half to two hours.

The pollen grain was intercepted at various altitudes. beginning with 10—40 m above the plantation and ending with 3000 m. the plane covering an area with a radius of 500—700 m. The glass slides with the pollen grain intercepted at a certain altitude were hidden away in a special receptacle. The count of the pollen grain was made immediately after landing and the number of grains per square centimeter calculated.

In order to determine the density of the pollen at various altitudes the slides with the pollen stuck on it were inspected in hundred places and the average density of the pollen per square centimeter deduced. The summary data on the density of the pollen at various altitudes is given in Table 1.

Since there were other plants besides the sugar-beet in blossom. the pollen of these plants was also intercepted.

In analyzing the mass of pollen floating in the air. it is necessary to deal with some of the flights at greater detail.

First Flight which lasted from 12.30 p. m. to 2.30 p. m. was made on June 30 when the sugar-beet field and the adjoining fields under white mustard (*Sinapis alba* L.). rape (*Brassica rapa oleifera*) and winter rye (*Secale cereale*) were beginning to blossom.

At altitudes varying between 200 and 1000 m the plane encountered a south-western wind a velocity from 11 to 18 m per second. The vehemence of the wind could be felt. Owing to the great disturbance of masses of air. pollen grain was discovered at a considerable height.

The flight showed the following picture (Table 2).

Third Flight was made in the morning hours of July 11 and lasted from 7.30 a. m. to 9.30 a. m.; the sugar-beet was in full blossom. the white mustard and rape were at the end of their blossom. while the blossoming of the rye had come to an end.

At an altitude of 1000 m an indifferent wind was blowing. During this flight the weather was relatively more calm than during the first flight (Table 3).

Table 1

Intercepted at Various Altitudes

in meters

400	500	750	1000	1500	2000	2200	2500	3000
547.9	378.24	159.09	67.12	2.60	2.60	2.60	—	—
12.32	8.52	3.58	1.51	0.06	0.06	0.03	—	—
661.35	632.30	302.55	78.78	3.05	3.8	—	0.65	—
10.24	9.78	4.68	1.23	0.05	0.06	—	0.01	—
696.6	499.24	391.5	199.45	59.15	6.5	2.6	1.95	—
11.3	8.10	6.32	3.22	0.94	0.15	0.04	0.03	—
599.2	503.6	277.3	117.45	4.55	1.3	2.6	1.3	—
11.52	9.68	5.32	2.25	0.08	0.02	0.05	0.02	—

The data obtained during the first and third flights point to the fact that the quantity of sugar-beet pollen varies at the various altitudes. During the third flight the quantity of sugar-beet pollen in the air was considerably greater than during the first flight. This is mainly because the ejaculation of pollen by the sugar-beet attains its greatest intensity in the height of blossoming.

Conclusions

1. The current opinion as to the poor dissemination of sugar-beet pollen through the agency of the wind has not been confirmed. The quantity of pollen in the air at considerable heights is larger or smaller depending on the period of blossoming and on meteorological conditions which vary in the course of the day.

2. The sugar-beet pollen is met with even at an altitude of 2200 m, which shows the possibility of disseminating pollen across long distances also in horizontal direction.

3. The greatest number of sugar-beet pollen grain has been observed at an altitude of 750 m. Higher up there is a sharp drop in the number of the grain.

Table 2

Percentage of Pollen of Various Plants Intercepted at Various Altitudes

Plant pollen	Altitude in meters											
	50	100	200	300	400	500	750	1000	1500	2000	2200	2500
Sugar-beet (*Beta vulgaris*)	32.26	26.7	42.2	37.0	37.0	37.6	47.7	87.1	75.0	25.0	50.0	—
White mustard (*Sinapis alba*)	34.65	47.0	47.4	43.8	46.3	50.9	47.7	10.6	25.0	—	50.0	—
Rape (*Brassica rapa oleifera*) . .	6.47	5.3	4.2	3.9	1.0	0.5	0.4	1.4	—	—	—	—
Winter rye (*Secale cereale*)	9 40	4.6	6.2	3.5	2.2	0.3	0.5	0.9	—	—	—	—
Pollen of other plants . . .	17.22	16.7	—	11.8	13.5	10.7	3.7	—	—	75.0	—	—

Table 3

Percentage of Pollen of Various Plants Intercepted at Various Altitudes

Plant pollen	Altitude in meters											
	50	100	200	300	400	500	750	1000	1500	2000	2200	2500
Sugar-beet (*Beta vulgaris*)	61.0	57.5	61.6	66.9	57.2	55.4	61.5	38.2	10.0	20.0	75.0	—
White mustard (*Sinapis alba*)	20.3	21.1	14.4	17.5	28.7	29.6	34.4	60.9	87.8	30.0	—	—
Rape (*Brassica rapa oleifera*)	4.7	7.6	5.9	6.6	2.6	2.0	0.6	0.9	2.2	10.0	—	—
Winter rye (*Secale cereale*)	—	—	2.7	—	0.9	—	—	—	—	30.0	—	—
Pollen of other plants	14.0	13.8	15.4	9.0	10.6	13.0	3.5	—	—	10.0	25.0	100.0

4. Investigations have shown that the larger pollen grains do not rise as high as do the smaller grains.

5. A very great ability of dissemination by air currents is displayed by white mustard pollen which has been found as high as 2000 and even 2200 m.

6. Lighter pollen grains of various other plants have been dedected at an altitude of 2500 m.

7. The fact that pollen of sugar-beet and other cross-pollinated plants is carried across long distances interferes seriously with their isolated cultivation.

8. Although sugar-beet pollen after having attained a height of 2200 m is likely to be carried away by horizontal currents of the upper strata of the atmosphere, nevertheless this should not be taken to mean that its transfer to short distances in the horizontal direction is less significant. Special investigations made by the writer of these lines in 1936, 1937 and 1938 on plantations occupying an area of from 60 to 95 hectares tend to show that the pollen is carried by horizontal air currents in the lower atmospheric strata for distances of from 4200 to 4500 m.

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POLYMORPHISMUS AND CENTERS OF FORMATION OF *PYRUS* SPECIES IN THE USSR

By G. A. RUBZOV

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The following 7 collective (Linnaean) species of pears are found in the USSR: *P. communis* L., *P. salicifolia* Pall., *P. elaeagrifolia* Pall., P. *heterophylla* Rgl. et Schmalh., *P. Korshinskyi* Litw., *P. syriaca* Boiss., *P. ussuriensis* Maxim., from 15 to 20 species in narrower delimitation, and a number of species of hybrid origin.

Since the time of Pallas (end the XVIII century) wild pears had attracted attention of many explorers of Crimea, Caucasus, Middle Asia and other regions of the USSR; however, despite the nearly one hundred and fifty years old history of studies, variation and processes of speciation in wild pears remained insufficiently known. The existing species and their geographic distribution are not yet fully known. Insufficient or even erroneous data on the distribution of species of *Pyrus* are contained in the floras for different localities. Thus, for example, *P. salicifolia*, *P. nivalis*, *P. amygdaliformis* are referred to as occurring in Crimea, whereas as a matter of fact they are absent there. The complex species *P. salicifolia*, *P. syriaca*, *P. communis*, *P. elaeagrifolia* are insufficiently studied in Caucasus. The genetical relations among peculiar species inhabiting Middle Asia as well as their connexions with species of adjoining provinces and the existence of eastern species are not yet cleared up.

An immense material in wild fruits has been accumulated during the last ten years by studies made at the All-Union Institute of Plant Industry. These studies have disclosed centers of striking polymorphism of pears in Transcaucasia and in Middle Asia. J. N. Voronov has described 3 new species in Transcaucasia, viz., *P. Raddeana*, *P. oxyprion* and *P. taochia*; 3 others were planned by him to be described. The following species and varieties were identified and described by M. G. Popov for Middle Asia: *P. Vavilovii* (*P. communis* × *P. Korshinskyi*), *P. heterophylla* var. *glabra* (*P. communis*×*P. heterophylla*), *P. Korshinskyi* var. *glabrescens*.

Closer investigation of the centers of variation found by expeditions has disclosed an astonishing polymorphism among pears of certain regions, which testified to a most intense formative process taking place there. The following regions belong to these centers: in Transcaucasia—south-western and southern Armenia (Daralagez, Megrinsky region), in Middle Asia—western Tian-Shan (Fergana mountains, Andijan mountains, Pekemsky range, upper part of the Chirchik river).

The material already collected makes it possible to speak of the existence in Armenia of at least 5 new species, aside from the two, *P. Raddeana* and *P. oxyprion*, earlier described by J. N. Voronov. I have distinguished *P. Voronovi*, *P. elata*, *P. Medvedevi*, *P. nutans*, *P. complexa* which are morphologically no less distinct than the species known previously. The number of biotypes of lower taxonomic order (varieties, forms) amounts to tens.

It is remarkable that such an immense variation is confined to a relatively limited area and to a definite zone. This is especially striking when the area in question is compared with other regions of Transcaucasia, for example with the whole of its northern part or with western Transcaucasia. Forests of wild pears occupy here tremendous areas, the common pear-tree being spread over a thousand kilometers, and yet its polymorphism is limited and variation of characters does not pass the limits of the species. The same uniformity is observed in Armenia proper, in its northern region, Delijan, Kirovokan and others which abound in wild pears.

On the other hand, in the above-mentioned regions of Southern Armenia—Daralagez and Megrinsky, for the space of not more than 200 km, in a 1500—2000 m zone in which there are dense woods of pears, at some spots on a few hectars or even in single small glades an astonishing variety of forms is opened to the eye; the differences among these forms comprise characters of the habitus of the leaves, fruit and flower and are of the same sort as those distinguishing species. In some spots there are as many forms as individuals, each of them possessing good taxonomic characters. To cite an example, among the 60 samples collected in the vicinity of the village Lichk of the Megrinsky region up to 20 well delimited forms could be distinguished.

The variety of forms concentrated in Southern Armenia is of particular interest with regard to its composition (species, varieties, etc.), to its variation and to genetical relations among the forms.

The most widely speread species here are *P. syriaca* and *P. salicifolia*. *P. communis* is rarely met and, besides, it is characterized by serrate leaves, closely resembling the type distributed over Talysh, and the Iran type possessing a long attenuate tip, rather than that common for other regions of Transcaucasia and, especially, for the Northern Caucasus, the type prevailing there being characterized by entire marginate leaves with short acute tips.

P. syriaca which is most widely spread over the zone at a 1500—2000 m altitude is distinguished by its extreme mutability, which is in accord with the data on changeability of this species in other provinces (Asia Minor—Boissier, Decaisne, Schneider). The variety of forms ranges from narrow-leaved, almost entire-marginate ones of the type of *P. Boveana* Decne, long-leaved crenate to broad-leaved of the type of *P. communis*. The intermediate form with lanceolate-cuneate leaves prevails; it is somewhat different from that typical of Syria and rather similar in the leaf shape to *P. amygdaliformis*, but different from it in crenate-serrate leaves, mace-like thickened peduncle, etc. Size, shape and quality of fruits vary to a highest degree. Forms which possess pear-like fruits on long peduncles are prevalent. *P. salicifolia* occupies the lower zone of the mountains and plateaux up to 1200—1500 m elevation, rising occasionally to 1900 m and more (Lake Seva, Megrinsky region). It is more uniform, but variations are also observed in shape, width and pubescence of the leaves, in length of the petiole, in size and shape of the fruit, in length of the peduncles and in the quality of fruits.

The above-noted variation in these species is mainly of quantitative nature. Although its range is wide, the specific type remains essentially intact.

Still clearer expressed and more interesting is the qualitative formative variation of pears in the same regions. Numerous intergradations in the kind

of differences and in the degree of expression of characters are observed; they range from insignificant ones relating to a single character, to large compounds of characters and complex neomorphisms. The following basic types of qualitative differences and morphogenesis may be noted.

1. Transgression of specific characters. Thus, the extreme forms of *P. salicifolia* from Vedi-lai are linked in the shape and pubescence of the leaves with *P. elaeagrifolia* so that the two are hardly distinguishable.

2. Variation in one or in a few characters of the kind of formation of a new character; the character is slightly expressed. The common forms of *P. syriaca* with slight pubescence, with sharp serrate or entire-marginate leaves with short petioles belong here.

3. Formation of new types representing a combination of characters of distinct species in unaltered or intermediate form. Most probably they are of hybrid origin. Numerous forms are found in which the participation of *P. syriaca* and *P. salicifolia* may be recognized.

4. New types which possess certain features similar to other species, but cannot be directly derived from them.

On the whole, the variety of forms is so great that the following species-cycles are mingled together: *P. syriaca*, *P. salicifolia*, *P. amygdaliformis*, *P. elaeagrifolia*, *P. communis*. Among many of widely distributed forms such are met with which approach those earlier described for neighbouring provinces (of Asia Minor) under the specific names *P. canescens* Spach, *P. Michauxii* Bosc., *P. persica* Pers., *P. Boveana* Decne, etc., and now included into *P. amygdaliformis* and *P. syriaca*.

Despite the presence of numerous transitional forms, delimitation of some of the forms is nevertheless clear with regard to a number of characters. These forms are found in isolated localities, i. e. they possess constancy and should be therefore considered large varieties or, following Darwin, «incipient species».

No less striking a variety of forms of pears which is hard to systematize is concentrated in the western Tian-Shan on spurs of the Fergana mountains, in the upper part of the Chirchik river and in other regions.

Among *P. communis* 15 types may be isolated. Variation of fruits is an extreme one. Forms with large fruits are very common. There are here innumerable intermediate forms connecting the following basic species: *P. communis*, *P. heterophylla*, *P. Korshinskyi* and *P. bucharica*. Aside from *P. Vavilovii*, *P. Vavilovii* var. *glabra*, *P. Korshinskyi* var. *glabrescens* described by M. G. Popov, *P. tomentosissima* closely related with *P. Korshinskyi* and with *P. bucharica* is distinguished by me; it differs from the latter in having strong felt-like white pubescence on leaves, shoots and buds, the densest among all the species of pears, large rounded, like in *P. elaeagrifolia*, buds.

What are the causes responsible for such a variation?

An analysis of the history of the floras of the regions, analysis of habitats and morphogenetic relations among species and forms do not leave any doubts as to its being on the one hand, a heritage of the remote past of the earth and result of recent processes of species formation, on the other.

Transcaucasia and Middle Asia are in the sphere of influence of the most ancient centers of xeromorphization and plant morphogenesis—of the Iran and Asia Minor. Some of the species living here bear features of relics. Such relics of the tertiary flora are *P. heterophylla* and *P. Boissieriana*. In these provinces species formation proceeded long since, took place natural selection, specialization in definite directions; all this was connected with a corresponding differentiation of the hereditary structure, with establishment of a relative constancy (homozygosity). *P. heterophylla*, *P. salicifolia* and *P. elaeagrifolia* belong to such specialized species.

In these very provinces, in mountain regions, conditions were preserved which most closely approached to the ancient ones. Preservation of the most primitive potentially heterogeneous types distinguished by intense morphogenesis is possible here. *P. Korshinskyi*, *P. syriaca*, *P. amygdaliformis*, *P. Boissieriana* and the local *P. communis* may be included here.

When one compares the immense variety of forms of pears in the zone of fruit-tree woods of Southern Armenia and of western Tian-Shan, the two provinces being characterized by a very broken relief and topographic diversity, with the monotony of forms in plains and in the zone of dense damp woods, one is convinced that forms of pears were and are developing just in the former zone.

This conclusion is in accord with data relating other plants. The last studies by Vavilov, by Tumanjan, by Sinskaja, Popov, Grossheim and others have demonstrated that Transcaucasia and Middle Asia are contemporary mighty centers of species formation of many plants and centers of progressive endemism (wheat, rye, alphalpha, *Rubus*, etc.).

By means of a detailed study of samples collected in Middle Asia I have demonstrated the existence of a large number of interspecific hybrids of all the species present here, of complex triple hybrids (*P. heterophylla*× ×*P. communis*×*P. serotina*, etc.). Their hybrid nature is proved by a number of characters and by a great number of intermediate forms. In particular, hybrids of *P. heterophylla* and *P. Korshinskyi* with other species may be well identified by seeds and by characters of leaves. Large size, coloration of seeds of the species and dissection of leaves typical of *P. heterophylla* are usually expressed in hybrids.

The same is observed in Southern Armenia (Daralagez, Megrinsky region), Daghestan, Azerbaidjan and in Crimea. Areas of different species are interlocking in all these regions: in western Tian-Shan of *P. heterophylla*, *P. Korshinskyi*, *P. communis* and of the cultivated *P. serotina*; in Armenia—of *P. salicifolia*, *P. syriaca*, *P. communis*; in Azerbaidjan and Daghestan—of *P. salicifolia* and of *P. communis*; in the Crimea—of *P. communis* and of *P. elaeagrifolia*. Maximum variety of forms is concentrated just at the point when different species meet with one another. In typical zones of distribution or habitats the species are, as a rule, biologically isolated owing to the discrepancy in times of blooming. This isolation is sufficient to keep adjoining species «pure» during thousands of years. However, under the conditions of relief in certain regions at a definite zone, as shown by various authors and by my own observations in Armenia, the periods of blooming of certain species may coincide so that hybridization becomes possible and actually takes place.

Experiments on artificial interspecific hybridization in pears conducted on a large scale at the Maikop and other stations of the All-Union Institute of Plant Industry testify to the ease of production of interspecific hybrids. Besides, facts of dominance of some of the species are being demonstrated, as well as peculiarities of ontogenesis which are of prime importance for breeding and for the study of phylogenetic relations within the genus *Pyrus*. To quote an example: dominance of *P. salicifolia* and of the Chinese species in crosses with the wild *P. communis* from Northern Caucasus, similar features in «juvenile» forms of many species, which point to genetical propinquity among the species. On the basis of an experimental study the true taxonomic position and the origin of forms are discovered, and they are of great importance both to the taxonomist and to the breeder.

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PLANT PHYSIOLOGY

WINTER WHEATS AS LAGGING BEHIND THE SPRING VARIETIES IN GROWTH INTENSITY WHEN SUBJECTED TO LOW TEMPERATURES

By I. M. WASSILIEV

(Communicated by N. I. Vavilov, Member of the Academy, 8. V. 1939)

Winter and spring wheats sown simultaneously by the end of a warm summer can scarcely be distinguished by their habitus when they have appeared above ground. Even the eye of an experienced breeder is not always able to discriminate them under those conditions. And it is only after the weather turns cold that the differences become apparent. In winter varieties the new forming leaves have shorter vaginae as well as shorter and more narrow blades. The bush grows loose and the shoots trail on the earth. The spring varieties, to the contrary, preserve their usual appearance, bearing broader leaves and looking erect and tall.

The changes in the habitus of winter wheats with the advent of cold weather in autumn doubtlessly bear upon the adaptability of these varieties to winter conditions. And of course their investigation will contribute much to inquiring into the nature of the frost resistance of winter cereals. In the present paper we refer to the results of a comparative study of growth intensity in the winter and in the spring wheats subjected to low temperatures.

The subject of investigation were ten, economically most important varieties. Hard spring wheats: *Melanopus* 069, *Hordeiforme* 010, *Hordeiforme* 0189 and *Mindum*; soft spring wheats: *Lutescens* 062 and *Caesium* 0111; winter wheats: Kooperatorka, Moskovskaya 02411, *Hostianum* 0237 and *Lutescens* 0329. Among spring wheats there were early and late varieties, *Lutescens* 062 furnishing an example of the most early, while at the other end of the scale was *Hordeiforme* 010. The winter wheats varied in both their quality, as winter crops and frost resistance. One of the lowest in both respects was Kooperatorka, while *Lutescens* 0329 ranked among the highest.

To secure a most uniform background for comparison, all the varieties tested were grown in the same vessel. This was usually a wooden box 40× ×40 cm broad and 20 cm deep, with a capacity of 20 kg soil. Each variety was represented in it by a row of 20 plants and there were no less than 10 such boxes in each of the tests, which were repeated several times. Two methods were made use of in the experiment. The first was to sow the plants at different dates and compare growth in the winter and the spring wheats under conditions of naturally changing temperature. The second consisted in simultaneous sowing and parallel growing of plants at different temperatures. All tests were carried on in Moscow in 1937—1938.

1. Winter and spring wheats sown in summer have grown under conditions

of high temperature. Sowing was made on May 28 in the open. High temperature favoured growth, and all of the plants were growing rapidly. As early as June 11, the main shoots bore 4 leaves each and had started tillering. The spring wheat had formed no stalks which in the earliest varieties appeared 2 days later. Temperature regime during the period from May 28 till June 11 was as follows: mean daily temperature mainly about +20, maximum +25°, minimum at night +13°.

To distinguish the winter from the spring varieties in this test by their outward appearance is a matter of considerable difficulty. Both kinds show a nearly identical habitus.

2. Winter and spring wheats have grown in hothouse under conditions of high temperature. Sowing made April 10 in a light hot room used for hothouse. The plants were placed near the window. The experiment lasted till May 5, the third leaf phase. The temperature varied from +16 to +18°C.

In this experiment it was quite impossible to discriminate the winter from the spring varieties by their habitus.

3. Winter and spring wheats sown early in spring have grown at a low temperature. The experiment was run parallel to that described above. Sowing made April 10 in the laboratory. April 13 when the seedlings had appeared above ground they were transferred in the open. Throughout April and the first 10 days of May the mean daily temperature was mainly below +10° C, while the minimum temperature was dropping below +5° at night. The experiment terminated May 17. Toward that time the spring wheats had not come into stalk.

The difference in the habitus of the spring and winter wheats is quite distinct in this experiment. The former stand erect and bear broader leaves, while the latter tend to a horizontal position and their leaves are small.

4. Winter and spring wheats have been suffering low temperature in greenhouse. Sowing made February 22 in a cool house. Till the end of the month and during the first 10 days of March the temperature was kept mainly at about +5°. During the rest of the experiment it varied between +5° and 10°, seldom rising higher. The experiment was stopped April 10, shortly before the early spring wheats came into stalk.

A very sharp difference is observed between the winter and the spring varieties, the former being considerably reduced in size*.

In some of the tests the growth of the roots was estimated along with that of the leaves. The mass of root system was found to correlated with the bulk of leaves. Both were lower in the winter than in the spring wheats, when growth was made at low temperature. Yet the difference in growth intensity between the winter and the spring varieties was not so sharply expressed in roots.

By comparing all the data obtained we reach the following conclusions:

1. Growth differences are observed between the winter and the spring wheats. If the temperature is high enough, in summer, the former make growth with the same intensity as do the latter. However, under reduced temperature, in autumn, they lag behind the spring wheats in growth. Their leaf vaginae are shorter and the leaf blades smaller.

2. The reduced growth intensity in winter wheats compared to spring wheats under low temperature conditions is their specific response to decrease

* It needs to be mentioned that such differences in growth as were observed between the spring and the winter varieties in the experiment will not appear on their growing in greenhouse under conditions prevailing in Moscow if they are sown earlier. Winter sowing, in November-January, brings about a depression of growth in spring wheats compared to winter varieties. The reason lies in the light intensity which is insufficient for the growth of spring wheats at that time.

Fig. 1. From left to right: spring wheats: *Melanopus* 069, *Hordeiforme* 010, *Hordeiforme* 0189, *Mindum*, *Caesium* 0111, *Lutescens* 062; winter wheats: Moskovskaya 02411, Kooperatorka, *Hostianum* 0237, *Lutescens* 0329. Photographed June 11, 1938.

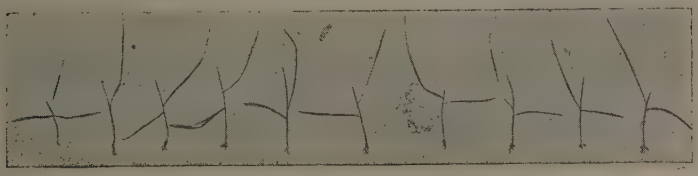

Fig. 2. From left to right: spring wheats: *Melanopus* 069, *Hordeiforme* 010, *Hordeiforme* 0189, *Mindum*, *Caesium* 0111, *Lutescens* 062; winter wheats: Moskovskaya 02411, Kooperatorka, Hostianum 0237, Lutescens 0329. Photographed May 5, 1938.

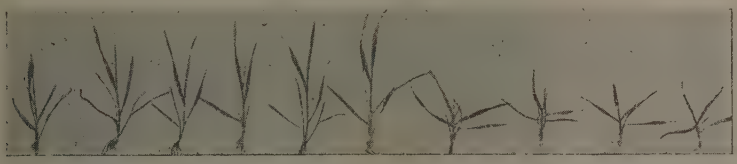

Fig. 3. From left to right: spring wheats: *Hordeiforme* 010, *Melanopus* 069, *Hordeiforme* 0189, *Mindum*, *Caesium* 0111, *Lutescens* 062; winter wheats: Kooperatorka, Moskovskaya 02411, *Hostianum* 0237, *Lutescens* 0329. Photographed May 17, 1938.

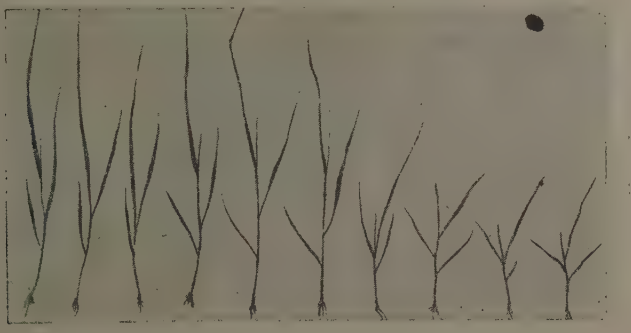

Fig. 4. From left to right: spring wheats: *Melanopus* 069, *Hordeiforme* 010, *Hordeiforme* 0189, *Mindum*, *Caesium* 0111, *Lutescens* 062; winter wheats: Kooperatorka, Moskovskaya 02411, *Hostianum* 0237, *Lutescens* 0329. Photographed April 10, 1938.

in temperature, which may be credited to play a certain rôle in their ability to withstand winter.

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PLANT PHYSIOLOGY

GRAFTING AS A METHOD OF CHANGING ALKALOID CONTENT IN PLANTS

By B. S. MOSHKOV and M. I. SMIRNOVA *

(Communicated by N. I. Vavilov, Member of the Academy, 8. V. 1939)

The question of the possibility for the alkaloid nicotine to pass through the place of junction from one of the grafted components to the other has been discussed by Linsbauer and Grafe (2) as well as by Meyer and Schmidt (3) for the case of closely related species of tobacco. The latter authors supposed the translocation of the alkaloid to be limited within a small zone around the contact. In our work we grafted lupinas with the purpose to make the alkaloids pass from the leaves of the stock to the tissues of the scion, as well as to obtain seeds of the latter with changed content of alkaloids. In undertaking such experiments we started from the fact that plant's metabolism is greatly dependent upon the synthetic activity of green leaves, by whose means the plant, according to the lively expression of Timiriazev, is performing its cosmic rôle. We were also guided by the indications of Michurin to the significance of leaves in grafting. The former works of one of the present authors (5,6) on the rate of photoperiods in the transition of plants from the vegetative to the reproductive stage and their frost resistance have shown that the processes taking place in the leaves essentially determine the morphogenesis of the growing points and the physiological condition of the plant as a whole. To be sure, it can hardly be doubted that the synthesis of alkaloids is also bound up with the lif eactivity of the leaves and that the amount of alkaloids in the tissues of one of the grafted components is therefore liable to change under the influence of the leaves of the other component. To verify this assumption was the object of our experiments in which grafting involved two species of lupines whose alkaloid content had been established previously by one of us (8). *L. albus* with its seed containing 3.41% of alkaloids and *L. angustifolius* with 1.21% were representatives of the alkaloid components, while another strain of *L. angustifolius* with only 0.019% was taken to represent the non-alkaloid (sweet) lupine. We were striving to bring about a maximum increase of alkaloids in the tissues and seed of the non-alkaloid lupine and reduce their amount in the alkaloid component as low as possible. The decrease of alkaloid content in lupine is of practical importance, while its increase cannot be said to have any, but should we succeed in controlling the process of alkaloid storage this might serve to develop a method suitable for working with plants which are valued on account of their alkaloid content, say cinchona.

* B. S. Moshkov—physiology, M. I. Smirnova—biochemistry.

Two methods of grafting were successfully used for the purpose. Plants of alkaloid or non-alkaloid lupine approaching the flowering stage served as stock, while the scion was either a top approaching the budding stage, or a top of young seedling plants from the same species. Four series of grafts were made: bitter lupine upon bitter lupine (control), bitter upon sweet, sweet upon sweet (control) and sweet upon bitter. In all cases the leaves of the stock were acting upon the tissues of the scion. Accordingly, the stocks were stripped of their shoots and all but the youngest leaves were removed

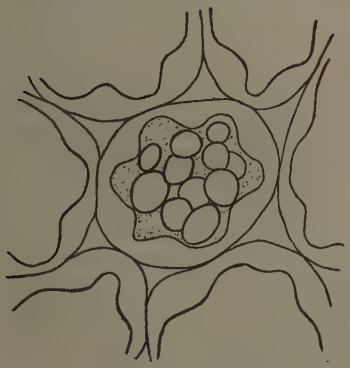

Fig. 1a.

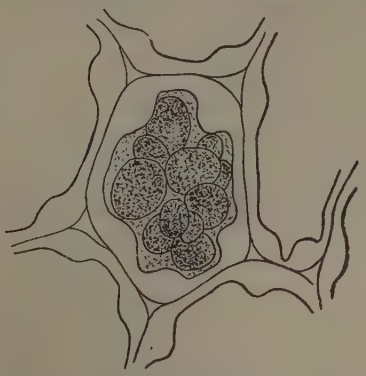

Fig. 1b.

Fig. 1c.

Fig. 1. *a*—sweet lupine cell (*L. angustifolius*) (0.019%); *d*—alkaloid lupine cell (*L. angustifolius*), (1.21%); *c*—cell of sweet lupine (0.019%) grafted upon a bitter one (1.21%).

from the scions. The estimation of alkaloids was done by the qualitative method worked out by Ivanov and Smirnova ([7]) for the purpose of breeding non-alkaloid lupines. Alkaloids were estimated both in the initial material, beginning with the seed, and in the scions several times during their development. There were many grafts made but most of the plants (especially sweet lupines) succumbed to the injury by fusariosis and mildew and only single of them produced ripe seeds. Alkaloid estimation showed that some few days after the graft had taken all the leaves of the sweet lupine scion turned alkaloid exceeding many times the control in alkaloid content (0.019%).

When the fruits made their appearance, the content of alkaloids in the leaves dropped and the pods grew bitter (alkaloid). Ripe seeds gave a distinct iodine reaction for the presence of alkaloids (Fig. 1).

Bitter lupines grafter on sweet ones failed to produce ripe seeds. But their content of alkaloids in the leaves was found to have decreased nearly three times compared to the initial bitter form of *L. angustifolius*.

It was therefore obvious that the sweet lupine increased its content of alkaloid under the action of the leaves of the bitter lupine, while the latter responded to the action of the leaves of the former by decreasing its alkaloid content.

The supposition suggested by the data obtained is that the seeds from such scions can produce plants whose content of alkaloids will not be peculiar to the original form. This was confirmed by other experiments in which

the same species of lupine were grafted when in the stage of seed germination. The seeds were made to germinate in the sand and as soon as the root emerged, the upper part of the embryo of one of the species was grafted upon the cotyledon of the other, whereby both species, the bitter and the sweet, served alternately as stock or as scion. The results of this operation was that the sweet lupine grew bitter, while the bitter lupine reduced its content of alkaloids as early as some few days after grafting, and these differences would not disappear during the further development of plants. The characteristic point to note here is that, the cotyledons being torn from the seed of the bitter lupine at once upon germination, the plants not only showed delayed growth, but they also contained less alkaloids compared to those in which cotyledons were not removed.

In the first experiment, when grafting was made at a later age, the new tissues and seeds changed their content of alkaloids under the influence of stock leaves which supplied them with all necessary organic substances. In the second experiment the presence or absence of sufficient amount of nutrient matter and of alkaloids in the cotyledons seems to have been a factor directing the processes which take place in the leaves and which determine the content of alkaloids in the plant. This suggests the processes going on in plants when pure preparations of alkaloids or initial products are introduced in their tissues (by injection or infiltration) to build up the alkaloid molecule. Such process, according to Klein and Linser ([1]), Blagoveshchensky ([4]) and Shmuk ([9]) lead to an enhanced storage of alkaloids.

From the rôle of the leaf life-activity in the storing of alkaloids by the seeds of lupine and the observed influence of the stock upon the scion we may infer that the content of alkaloids in the plants can be controlled by means of grafting.

The method described must of course be modified if it has to be applied to other alkaloid plants, cinchona in particular, whereby their vegetative propagation and not by seeds must be taken into consideration. But the very possibility to change by experiment and therefore to increase the amount of alkaloids is of great practical importance.

The most interesting point to clear up in the next investigations is the question as to how long this after-effect of grafting which essentially takes its origin in the action of leaves of one of the components upon the tissues and metabolism of the other, will last in the plants on their vegatative or seed propagation.

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EXPERIMENTAL MORPHOLOGY

INFLUENCE OF SUPRARENAL CORTEX ON THE SEXUAL APPARATUS

1. On the gonadotropic action of the cortex of suprarenal glands of cattle

By V. R. KLIACHKO

(Communicated by I. I. Schmalhausen, Member of the Academy, 13. V. 1939)

The alterations arising in the sexual apparatus in connexion with hyperplasy or tumors of the cortex of suprarenal glands have been long since described by a number of clinicians. Most frequently tumors of the suprarenal cortex induce virilism in female and on much rarer occasions feminization in male patients. In children the intensification of the function of the suprarenal cortex results usually in an early sexual development, heterosexual development being more frequent in girls and the isosexual one in boys.

The present study is devoted to the investigation of the gonadotropic function of the suprarenal cortex which it is presumed to possess on the ground of the early sexual maturing of children suffering from tumors of the suprarenal glands.

Experiments with Transplantation of Suprarenal Glands to Infantile Mice. This experiment included 7 litters of white mice, a part of each litter serving as controls. The total number of experimental animals was 26, that of controls—16. The suprarenal glands were excised from adult female mice and transplanted under precautions of sterility under the skin of the back to infantile mice, bits of muscle being grafted in the same manner to the control infantile mice. In 4 litters two suprarenal glands were transplanted to each mouse, in 2—four and in the 7th—six. The mice were killed at the age of 21—23 days. In the females ovaria and uterus were subjected to histological examination, in the males—testes and prostate; besides this, we examined in every animal its thyroid gland and the transplanted suprarenal glands. A certain stimulation of the sexual apparatus was observed only in the litter in which 6 suprarenal glands were transplanted to each animal. In the ovaria of the experimental animals of this litter larger follicles were found than in those of the controls, and they had already small cavities filled with follicular liquid. The epithelium of the uterus was higher in the experimental animals than in the controls. The testes of the experimental mice did not differ in any essential point from those of the control ones. The grafts showed degenerative alterations of the cortical cells. The thyroids of most experimental mice showed signs of a slight hypofunction.

It was impossible to draw any definite conclusions concerning the stimulating influence of the suprarenal cortex on the sexual apparatus of infan-

tile mice. However, the slight signs of such stimulation under the influence of the transplantation of suprarenal glands which we had observed induced us to suppose that it might prove possible to resolve this problem with success by introducing greater amounts of the cortex, i. e. by introducing cortical extract.

Experiments with Introduction of Suprarenal Cortex Extract Obtained from Cattle. The extract was prepared according to Hoffmann [7]. Cortex of the suprarenal glands of cattle and pigs was carefully separated from the core, ground on a meat-grinder and defatted with acetone. The powder thus obtained was dried and subjected to extraction with $n/50$ NaOH, the alkaline extract was neutralized and precipitated by 96° alcohol. The precipitation was centrifuged, washed with absolute alcohol and ether. The alcoholic powder was subjected to extraction with water, and this water extract was administered to the experimental animals. At various dates I have prepared in this manner 6 lots of the extract, treating for this different quantities of cattle suprarenal glands varying from 500 g to 1.5 kg.

Treatment of Infantile Rats with the Extract. The extract thus obtained was tested on 52 female infant rats weighing from 30 to 40 g. The extract was injected to the rats for 2 days in doses of 0.5—1.0 cm^3 three times daily. From all the rats vaginal smears were taken after the opening of the vagina. 76, 100 or 120 hours after the first injection the rats were killed. The ovaria and the uterus were weighed out and fixed for histological examination. Besides the ovaria I examined histologically also the thyroid glands and in several cases the uterus. 30 control rats (a part of these belonged to one litter with the test ones) were killed when they reached the weight of 45—51 g (the weight of the test animals on the day they were killed). The weight of the ovaria of the test animals varied between 7 and 14 mg, the average weight being 10 mg, the weight of their wombs varied between 20 and 40 mg, the average being 27 mg. In the control animals we never observed the opening of the vagina. The ovaria and thyroid glands of 20 control rats were examined histologically, in several cases we examined also the wombs. The control series was utilized for comparison with all the series of experimental female rats.

Results of the Experiments. Under the influence of suprarenal cortex extract (from 200 to 700 mg of alcoholic powder, equivalent to 24—30 g of the cortex) the vagina opened and the rut began in the experimental rats. The earliest time at which the opening of the vagina was observed was 56 hours after the first injection, the latest—94 hours. The weight of the ovaria in the rats showing oestrus varied between 12 and 22 mg, the average being 16 mg; the weight of the uterus varied between 46 and 166 mg, the average weight equalizing 77 mg (cf. Table). The histological examination of the ovaria revealed the presence of large-sized mature follicles with great cavities filled with follicular liquid (in the control animals the follicles were of smaller size and their cavities quite small). No corpora lutea were found in any of the cases. The wombs showed alterations characteristic of the oestral phase. The examination of the thyroid glands did not provide grounds for any definite conclusions.

The experiment described above showed that 1 kg of the suprarenal cortex contained an amount of gonadotropic substances of the factor A type, varying between 11 and 41 rat units.

In a part of both control and experimental rats which had been treated with suprarenal cortex extract we investigated the gonadotropic activity of the hypophyses (by the method of transplantation to infantile mice or by injecting a suspension of ground hypophyses). The activity of the hypophy-

ses was evaluated by the weight of the uterus and ovaria of the mice. The results obtained did not show any significant differences between the controls and experiments, this providing grounds for presuming that the extracts action is not exercized via the hypophyses. Special experiments on hypophysectomized rats are to be arranged for the elucidation of this question.

Treatment of Mice with the Extract. The experiment was arranged on 10 mice weighing 6—8 g. The dose of the extract equalized 100—200 mg of alcoholic powder, i. e. 0.5—1 rat unit. The extract was administered to the mice for 2 days, 0.4 cm^3 being injected 3 times daily. Vaginal smears were taken for examination from every mouse after the opening of the vagina. 100 hours after the first injection the mice were killed. The control group consisted of 20 mice which were killed when their weight reached 7.8—10.8 g (the weight of the test mice in the day they were killed). The weight of the ovaria varied between 2 and 3 mg, the weight of the uterus—between 7 and 11 mg. The histological picture of the ovaria and uterus was that usually observed in infantile mice. The 1 rat unit dose provoked the appearance of oestrus. The weight of the ovaria varied between 4 and 6 mg, that of the uterus—between 17 and 25 mg. Histological examination revealed in the ovaria of the test mice maturer follicles than in the control ones. «Blutpünkte» and corpora lutea were absent. The picture of the womb corresponded to the oestral phase.

Introduction of Cattle Suprarenal Cortex Extract

	Number of animals	Variation of weight of ovaria (mg)	Average weight of ovaria (mg)	Variation of weight of uterus (mg)	Average weight of uterus (mg)
Control group of rats .	30	7—14	10	20—40	27
Experimental group of rats	19	12—22	16	46—166	77
Control group of mice .	20	2— 3	2.4	7— 11	9
Experimental group of mice	5	1— 6	4.9	17— 25	19

Note. The above figures are those obtained after the administration of an oestrus-provoking dose.

Experiments with the Introduction of Pig Suprarenal Cortex Extract. This extract was tested on 6 infantile rats, weighing from 34 to 41 g. Doses corresponding to 37.5—45 g of suprarenal cortex provoked oestrus. The weight of the ovaria was 12, 15 and 17 mg, that of the wombs—54, 62 and 64 mg. Histologically the ovaria showed larger follicles, the picture of the uterus was the one corresponding to the oestral phase.

Introduction of suprarenal cortex extract prepared from the glands of rabbits (14 animals) and rats (29 animals) did not supply clear results, possibly owing to the insufficient amount of the extract.

Testing of the Suprarenal Cortex for Oestrogenic Activity. Though the chemical treatment involved in the preparation of the extract excluded the possibility of the presence of folliculin in it, yet it seemed of interest to test the extract for its oestrogenic activity, since the introduction of polypeptides may provoke in the genitalia of rats alterations usually due to the female sexual hormone (1). The

experiment was arranged on 5 castrated mice in which a continuous dioestrus was observed during 6 weeks. The introduction of the extract (2 to 4 rat units) did not give rise to any alterations in the picture shown by vaginal smears. Consequently, under the conditions of our experiment the extract did not possess an oestrogenic activity.

Thus, from our experimental data it was obvious that the cortex of the suprarenal glands of cattle and pigs contained gonadotropic substances, however, the question whether these substances are produced or only accumulated in the suprarenal cortex remained open. Hoffmann has shown that there are no gonadotropic substances in the cortex of the suprarenal glands of castrated animals, when the organism contains an excess of gonadotropic substances and, consequently, when the conditions for their accumulation are most favourable. Besides this, according to Hoffmann, the gonadotropic substances of the suprarenal cortex differ from the gonadotropic substances of the hypophysis in that they are capable of ultra-filtration.

To clear up the question concerning the accumulation of gonadotropic substances I investigated the content of gonadotropic substances in the ovaria of cattle, since it was but natural to suppose that, if gonadotropic substances accumulate in endocrine glands, they must be present also in the ovarium, i. e. in an organ which directly depends on their action.

Determination of gonadotropic Substances in the Ovaria of Cattle. An ovarian extract prepared according to the method of obtaining gonadotropic substances from the suprarenal cortex which we described above was tested on 8 rats weighing from 33 to 41 g. The highest doses of extract used corresponded to 116 and 162 g of ovaria, i. e. were much higher than the doses of suprarenal cortex extract. In no case did the introduction of ovarian extract give rise to an early sexual maturity; the vagina remained closed; the weight of the ovaria varied between 9 and 13 mg, that of the wombs—between 24 and 31 mg. Histological examinations of the ovaria revealed no signs of their stimulation, either.

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ZOOLOGY

A NEW GENUS OF SAND FORAMINIFERA FROM THE ARCTIC SEAS

By Z. STSCHEDRINA

(Communicated by A. A. Borissiak, Member of the Academy, 8. V. 1939)

When treating the rich materials on Foraminifera put at my disposal, a most interesting form was found which owing to its main morphological features should be referred to a new genus. The present paper deals with the description of this form.

Arenosphaera, gen. now. Test free or attached consisting of from one to several (up to 3) nearly spherical chambers. Test wall built of large coarse sand grains disposed in one layer and adhering to one another so loosely that pore-like spaces exist between them.

The test possesses no other apertures. The most interesting feature of this form is the presence of large openings in the test walls; they seem to perform the functions of the absent aperture. The thick coarse-grained test is perforated all over its surface by numerous openings of various size and form depending upon the form of the sand grains by which they are delimited. The presence of these pores together with other peculiarities have induced me, as indicated below, to consider this form as representing a new genus.

According to the main structural characteristics this form belongs to the Fam. *Saccamminidae*, Subfamily *Psammospherinae* [v. Dr. Cushman's system (1)] which comprises the agglutinated tests lacking a «definite» aperture. This subfamily includes the following genera: *Psammosphaera* F. E. Schultze (1875). *Sorosphaera* H. B. Brady (1879) and *Storthosphaera* F. E. Schultze (1875). From the genus *Psammosphaera* our form differs by its relatively larger number of chambers and the structure of its test walls which in *Psammosphaera* consist of a thin inner pseudochitinous layer, while externally it is built of closely cemented sand grains, sponge spicula etc. In *Arenosphaera* a pseudochitinous layer has not been found and the test wall consists, as said above, of very loosely cemented particles. Neither is our form identical with the one or the other of this two genera—*Sorosphaera* and *Storthosphaera*—differing from both of them by the absence of the inner pseudochitinous layer *(Sorosphaera)*, the nature of the particles constituting the wall and the structural characteristics of the test wall.

Moreman (2) made of these three genera, adding to them two new genera *Stegnammina* Moreman (1930) and *Raibosammina* Moreman (1930) he had described, a new family—*Stegnamminidae*, its forms being characterized by the absence of an aperture. In my opinion the isolation of such specific forms from the large and very variously constituted family *Saccaminidae* into a separate taxonomic group is fully justified. The new genus here described should be referred to the Fam. *Stegnaminidae* as it presents the characteristic peculiarity of this family—absence of an aperture.

Arenosphaera perforata, sp. nov. (Fig. 1)

Test large, for the most part free, occasionally attached. Consists mostly of one large chamber of nearly spherical form, but tests constituted of two or three chambers may also occur, in which case the chambers lie in one straight line, each of them embracing the next in order. The chambers differ sharply as to size, the terminal chamber being about twice as large as the initial one. All the chambers retain a nearly regular spherical shape; they are connected with one another by broad round openings. The external sutures between the chambers are rather indistinct, they are slightly depressed. The test wall is thick and has a coarse surface. It is constituted of angular sand grains, fragments of large Foraminifera and sponge spicules. The separate components, the sand grains especially, are regularly orientated, their coarse surface turned outwards, while the smooth surface is turned inwards. The particles adhere loosely to one another, the spaces between them representing pores of various shape and size (Fig. 2).

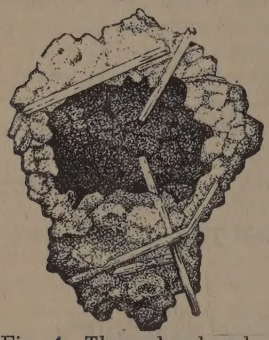

Fig. 1. Three-chambered specimen of *A. perforata*; the last chamber is broken. Anterior portion of the wall removed.

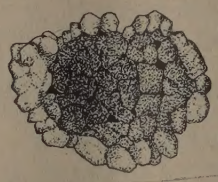

Fig. 2. A portion of the test wall of *A. perforata*. × 40.

Colour of the test—grey	
Length of a three-chambered test	—6 mm
Longitudinal axis of the initial chamber	1 »
» » » » second »	2 »
» » » » third »	3 »
Transverse » » » initial »	0.75 mm
» » » » second »	2.5 »
» » » » third »	3 »
Length of a two-chambered test	4 mm
Longitudinal axis of the initial chamber	1 »
» » » » terminal »	3 »
Longitudinal and transversal axis of a one-chambered test	3 »

Distribution. In benthonic samples—mostly in small numbers associated with large sandy Foraminifera (*Hormosina globulifera* H. B. Brady, *Saccorhisa ramosa* H. B. Brady and others) in the Kara Sea at a depth of from 92 to 342 m in oozy ground or in ooze with sand and gravel; bottom water temperature—1.84 to 1.88° C. The form was found also in the collections of the «Sadko» expedition 1935 in the Greenland Sea (700 m) and NNW off the northern point of Spitzbergen (670 m) as well as in the Polar basin (698 m). Tests very brittle but easily identified even in small fragments because of the characteristic structure of the test wall.

This Foraminifer owes its name to the fact that its most frequently occurring form is a nearly regular sandy sphere perforated by numerous openings.

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